Who Works for Whom? Worker Sorting in a Model of Entrepreneurship with Heterogeneous Labor Markets*

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Abstract

Young and small firms typically hire younger workers that come from nonemployment, and provide these workers with lower earnings compared to established firms. To understand these facts, a dynamic model of entrepreneurship is constructed, where individuals can become entrepreneurs, or work in corporate or entrepreneurial sector. Sectoral differences in production technology, financial constraints, and labor market frictions lead to sector-specific wages and worker sorting into the entrepreneurial sector by productivity and assets. Individuals with lower assets tend to accept jobs in the entrepreneurial sector, an implication that finds support in the data. The analysis indicates that labor market frictions are important in the model's ability to generate worker sorting and to match the key features of the entrepreneurial sector.

Keywords: Entrepreneurship, borrowing constraints, worker sorting, labor market frictions, employment dynamics

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1 Introduction

Job creation by entrepreneurs is an important component of employment dynamics in the United States. In a typical year, new firm startups account for about 3% of total employment but almost 20% of gross job creation.[1] The jobs entrepreneurs create, however, may not always be the most desirable ones. Entrepreneurial firms, which are generally thought of as privately-owned young and small firms, provide lower earnings on average to their workers compared with more established, older, and larger firms.[2] They also tend to hire disproportionately from the pool of workers who are young and nonemployed.[3] Jobs in entrepreneurial firms may therefore serve an important section of the labor market by providing employment opportunities for those who would otherwise be nonemployed or those who have to wait too long for a higher-paying job in an established firm. Despite the differences in worker characteristics and earnings across the two groups of firms, the mechanisms by which workers sort across entrepreneurial versus other firms, and how this sorting is influenced by various labor market and financial frictions, remain relatively less understood.

There is also a growing concern about the current state of entrepreneurship in the United States, fueled by the long-decline in business startups and diminished business dynamism.[4] These trends call for a better understanding of the connection between the supply of entrepreneurs and the characteristics of the labor market for entrepreneurial firms. The long decline in entrepreneurship has implications for those who tend to work for entrepreneurs, and conversely, changes in the labor market over the past few decades have consequences for entrepreneurs' ability to hire and retain workers, and hence, for their cost of entry and doing business. The interrelated nature of the supply of entrepreneurs and the supply of labor to them leads to several questions. What kind of individuals choose to work for entrepreneurs, and why? How do financial and labor market frictions affect the decision to become an entrepreneur and to work for one? Which type of friction is more critical in the match between entrepreneurs and workers? How much does the degree of frictions matter? These questions demand a framework where individuals face not only the decision to become entrepreneurs, but also the decision to work for entrepreneurial versus other firms.

This paper develops a model to understand who becomes an entrepreneur and what kind of

[1] At the same time, about 40% of the jobs created by startups disappear due to failure within 5 years of entry. See Haltiwanger, Jarmin, and Miranda (2013).

[2] See, e.g., Brown and Medoff (1989) for the connection between firm size and earnings. Brown and Medoff (2003), Kölling, Schabel and Wagner (2002), and Dinlersoz, Hyatt, and Nguyen (2013) document, among others, the connection between the age of an establishment or a firm, on the one hand, and the average earnings of workers, on the other.

[3] See, e.g., Ouimet and Zarutskie (2014) and Goetz, Hyatt, McEntartfer, and Sandusky (2015).

[4] See, e.g., Decker, Haltiwanger, Jarmin, and Miranda (2014a).

workers sort into entrepreneurial firms in the presence of search frictions in the labor market and financial constraints for entrepreneurs. In the calibrated model's equilibrium, individuals with high labor productivity and lower assets tend to sort into the entrepreneurial sector. The empirical findings support the implication of the model on worker sorting by assets. The analysis also explores in detail the mechanisms in the model that generate entrepreneurship and worker sorting. While several features of the model matter for equilibrium allocations, labor market frictions in particular play a key role in generating the extent of entrepreneurship and the degree of worker sorting into entrepreneurial firms observed in the data.

In the model, individuals differ in wealth, entrepreneurial ability, and worker productivity. Each individual can become an entrepreneur, or work in one of the two sectors: entrepreneurial and corporate–a label for the set of firms that don't face the constraints entrepreneurial firms do. The constraints entrepreneurs face are of two types. The entrepreneurial production is subject to diminishing returns that arise from the limits to entrepreneurs' span-of-control. In contrast, firms in the corporate sector can scale up production without such restrictions. In addition, entrepreneurs can borrow only up to a limit to operate their businesses–a constraint that does not apply to corporate sector firms.

The choice to become an entrepreneur and to work for one are endogenously determined, along with the price of labor entrepreneurs face. The match between workers and firms is subject to frictions in the labor market. Not all nonemployed individuals who look for a job can find one, and workers can be separated from their employers involuntarily, in addition to voluntary separations. These labor market frictions are allowed to vary across the two sectors. Job offers arrive at different rates, and involuntary separations occur with different probabilities. The differences across the two sectors in production technology and labor market frictions together lead to divergence in sectoral wages per unit of worker efficiency. The wage differential, combined with the heterogeneity in worker productivity and wealth, implies that individuals who choose to work sort across the two sectors based on both productivity and wealth.

The model outlined above is related to recent models on entrepreneurship.[5] As in these models, the model can account for the observed fraction of entrepreneurs in the population, as well as the distributions of wealth for entrepreneurs and workers. What distinguishes it from others, however, is the presence of sector-specific labor market frictions and prices of labor. These features together generate employment shares, worker earnings, and worker flows for the two sectors that are consistent with the observed counterparts. They also lead to differences in the types of workers the two sectors attract. The model's equilibrium entails both a lower wage per unit of worker efficiency and lower average worker earnings in the entrepreneurial sector

[5] See, among others, Quadrini (2000), Cagetti and De Nardi (2006), Kitao (2008), Buera and Shin (2011), Buera, Fattal-Jaef, and Shin (2015), and Bassetto, Cagetti, and De Nardi (2015).

relative to the corporate sector. The earnings premium in the corporate sector emerges due to a combination of factors. One factor is that job offers arrive at different rates from the two sectors. Other factors are the decreasing returns to scale and borrowing constraints in the entrepreneurial sector, which limit the ability of the entrepreneurs to offer high wages. As a result, the wages per unit of worker efficiency are not necessarily equalized across the two sectors. Given this wage gap, on average more productive individuals choose to work for the corporate sector, resulting in a corporate sector earnings premium.

The model provides an answer to the central question of who works for whom. Workers in the entrepreneurial sector tend to be less productive and have lower assets, compared to those in the corporate sector. The asset differential is in part a consequence of the fact that individuals who work in the higher-wage corporate sector can accumulate more wealth over time than their counterparts in the entrepreneurial sector. However, a selection effect is also present: individuals who take jobs in the entrepreneurial sector tend to be less wealthy even *at the time* they take these jobs. In other words, the wealth and productivity differences across the two sectors also apply to individuals who are in their first period of employment. Nonemployed individuals with a job offer from the entrepreneurial sector have to decide whether to reject this offer and wait for an offer from the higher-wage corporate sector.[6] Individuals with lower levels of savings and productivity prefer to take jobs in the entrepreneurial sector rather than waiting. This sorting of individuals emerges in the model in the absence of any inherent preference for working in entrepreneurial firms, or any form of compensation, other than wages, such firms can provide.

The model's prediction that workers with lower assets tend to more readily take jobs in entrepreneurial firms is taken to data. The test of this prediction requires data not only on individuals' assets, but also on their employment choices and the characteristics of their employers. While data on worker assets is available from a variety of sources, measuring workers' assets by employer type (e.g. employer size or age) and especially at the time when they start a job is more challenging. The analysis combines data on workers' net worth from the Survey of Income and Program Participation (SIPP) with the data at the worker-job level from the Longitudinal Employer-Household Dynamics (LEHD) program that captures employer characteristics and workers' job transitions. The findings suggest that individuals who work in younger and smaller firms tend to have fewer assets than their counterparts in older and larger firms. Furthermore, individuals who take jobs in younger and smaller firms also tend to be less wealthy around the time they take these jobs compared to those in older and larger firms.[7] These findings support

[6] The role of asset heterogeneity in generating differential labor market outcomes has also been explored recently by Eeckhout and Sepahsalari (2014), and Herkenhoff, Phillips, and Cohen-Cole (2015). A similar mechanism is also at work in theoretical study of Browning, Crossley, and Smith (2007). While these studies include directed search in labor markets, they do not model an entrepreneurship versus work decision.

[7] Herkenhoff, Phillips, and Cohen-Cole (2015) provide empirical evidence that, among workers who undergo an

the predictions of the model on worker sorting based on assets.

To understand the mechanisms behind worker sorting and equilibrium allocations, a series of experiments are carried out to isolate the roles of the model's key elements. One main feature of the model is the presence of distinct sectoral wages for labor, in contrast to a uniform wage in many previous models of entrepreneurship. To isolate the effect of this feature, versions of the model is studied where the entrepreneurial sector wage is constrained to be equal to the corporate sector wage. Despite the uniform wage, worker sorting by productivity and assets prevails. In the uniform wage environment, job finding and separation frictions play an important role in maintaining worker sorting. These frictions also matter for the model's ability to match key facts about the entrepreneurial sector.

The extent of the borrowing constraint for entrepreneurs, another key element in the model, also matters. When entrepreneurs are allowed to borrow more than the typical limit imposed in the literature (50% of an entrepreneur's total assets – see, e.g., Buera and Shin (2011)), they are able to operate larger businesses and actually pay higher wages than the corporate sector, in a reversal of the baseline model's wage ranking. However, even with the entrepreneurial sector's advantage in wages, workers with higher assets on average continue to sort into the corporate sector. Labor market frictions in the entrepreneurial sector again emerge as a key ingredient in maintaining worker sorting and equilibrium allocations in the model with higher borrowing limit.

Another important feature of the model is that at the time of the entrepreneurship decision entrepreneurs face uncertainty about their initial ability draw. Removal of this uncertainty results in a higher threshold of selection into entrepreneurship. Only the individuals with relatively high entrepreneurial ability become business owners and they operate larger businesses on average. The entrepreneurial sector is then able to pay a higher wage than the corporate sector, as in the case of the relaxed borrowing constraint. However, the baseline model's worker sorting result still emerges under no uncertainty. Once again, labor market frictions turn out to be crucial in maintaining worker sorting and equilibrium allocations.

The analysis in this paper is also relevant for understanding certain aspects of the decline of entrepreneurship in the United States. Recent research has documented a decades-long decline that accelerated during the Great Recession.[8] The share of young employers in the population of

involuntary job separation, those with more credit card debt spend less time in unemployment and accept lower wages, which is consistent with the evidence here that young and small (i.e., lower-paying) firms disproportionately hire workers with lower assets.

[8]Recent work on this decline include Siemer (2014), Pugsley and Sahin (2015) and Karahan, Pugsley, and Sahin (2015). These studies focus mainly on the decline in firm startups or age zero firms (a flow measure), which are a subset of the broader set of entrepreneurial firms (a stock measure) in the economy at any point in time, and abstract from labor and financial markets.

firms has been falling, and workers are increasingly employed in older firms.[9] The new businesses that have formed recently tend to create fewer jobs, and the decline of business startups explains part of the decline in worker reallocation rates.[10] Furthermore, average worker earnings in young firms have fallen, relative to that in old businesses. These findings highlight the connection between the supply of entrepreneurs and the labor market for entrepreneurial firms. A fall in the supply of entrepreneurial firms and their wages can lead to more unemployment and lower earnings for individuals who typically match with such firms. These effects can reinforce earnings and wealth inequality in the economy, since these individuals tend to be less wealthy to start with, as the analysis here indicates. Moreover, as the dynamics of the labor markets change in response to the shifts in worker demographics and mobility and increasing dominance of more established firms, some entrepreneurial firms may face increasing frictions in attracting and retaining workers, leading to higher costs for entrepreneurs and a further decline in entrepreneurship. Studying these effects require a framework with heterogeneous labor markets with frictions, where the entrepreneurship decision, wages in entrepreneurial firms, and the choice of employment in the entrepreneurial sector are jointly determined. The model considered here provides one such framework.

The rest of the paper is organized as follows. The next section documents some key facts about entrepreneurial firms. Section 3 introduces the model, followed by its baseline calibration in Section 4. The properties of the baseline model are discussed in Section 5. Section 6 focuses on exploring the role of some of the model's key elements in determining equilibrium allocations and worker sorting. Section 7 offers empirical evidence on the predictions of the model on worker sorting by assets. Section 8 concludes.

2 Some Observations on Entrepreneurial Firms

This section documents some empirical findings about entrepreneurial firms to motivate the model and its analysis. A fundamental question is what constitutes an entrepreneurial firm. Entrepreneurial firms are often thought of as privately-held, young, and small firms (in terms of employment).[11] Although there are some young and small firms that may not be entrepreneurial in nature (e.g. new businesses created by established firms), and some entrepreneurial firms that are young but large, firm age and size are frequently used to approximate the population of

[9] See, e.g., Decker, Haltiwanger, Jarmin, and Miranda (2014a) and Hathaway and Litan (2014).

[10] See, e.g., See Sedlacek and Sterk (2014) and Hyatt and Spletzer (2013).

[11] The authors' calculations based on the 2007 Survey of Business Owners confirms this view of entrepreneurial firms. Of young businesses (less than 5 years old), 69.1% are owned by households, while such household-owned businesses account only 45.7% of the employment in these young firms. Of small firms (less than 50 employees), 88.4% are owned by households, and such businesses account for 40.0% of the employment in small firms.

entrepreneurial businesses. However, alternative definitions can also be provided.

Table 1 presents various definitions of entrepreneurial firms and some key statistics associated with these firms for the year 2000. In all definitions, non-employer businesses are excluded, as the focus is on entrepreneurs who create jobs. In addition, each firm is assumed to have a single owner.[12] Assuming that the pool of potential entrepreneurs is the population of males aged 15-64 years in 2000, the fraction of entrepreneurs in the economy can then be calculated as the number of entrepreneurial firms divided by that population.[13] One way to define entrepreneurial firms is to apply various age and size criteria to the universe of employer-businesses in the U.S. Census Bureau's Longitudinal Business Database (LBD). Based on these criteria, Table 1 reveals that the fraction of entrepreneurs ranges from a rather conservative estimate of 1.7% to a less stringent one of 5.8%. Alternatively, one can define entrepreneurial firms as those that are not publicly owned and that have indicated some ownership demographics in the U.S. Census Bureau's Survey of Business Owners (SBO). This approach yields an estimate of 6.0%. To provide another estimate, one can use the responses to the question regarding employer-business ownership in the Survey of Income and Program Participation (SIPP). The estimates in this case vary from 2.3% to 2.9%. Table 1 also indicates that employment share of entrepreneurial firms varies between 3.6% to 44.0% across various definitions. These definitions also imply a non-entrepreneurial firm average earnings premium in the range 16.6% to 49.8%.[14]

Consider now some important differences between entrepreneurial and other firms relevant for the analysis in this paper. Suppose an entrepreneurial firm is defined as a firm that is at most 5 years old or a firm that has at most 20 employees – the findings are robust when different definitions of an entrepreneurial firm in Table 1 are used instead.[15] As Table 2 indicates, entrepreneurial firms offer lower earnings to their workers on average. In 2000, the median of the average worker earnings for young firms was about 85% of that for other firms, whereas by 2012 this figure dropped to about 75%. For small firms, the corresponding figures are 73% and 76%. The documented gap in average earnings is consistent with the broader empirical literature on firm age and size premia in worker earnings.[16]

[12] The datasets used to construct Table 1 have different definitions of business ownership. For example, household surveys count business owners who may operate multiple businesses, while business-level surveys do not identify which businesses have owners who also own other businesses.

[13] In 2000, the population of males aged 15-64 years amounted to approximately 93 million based on U.S. Census Bureau's American Fact Finder.

[14] The average worker earnings premium is defined as excess average worker earnings in non-entrepreneurial firms expressed as a percentage of the average worker earnings in entrepreneurial firms.

[15] For instance, using a 10-year threshold for an entrepreneurial firm leads to similar qualitative results.

[16] Brown and Medoff (2003) find that average worker earnings are lower in younger firms in a sample of U.S. firms. This finding has repeatedly emerged in studies using a variety of datasets. For instance, Kölling, Schabel and Wagner (2002) largely confirm Brown and Medoff's (2003) findings using data that links establishments to

Table 2 also contains information on the relative frequency and size of entrepreneurial firms. Young firms accounted for nearly half of all firms in 1987, but only one-third in 2012. Compared to their relative frequency in the firm population, young firms account for a relatively small share of total employment: nearly one-fifth in 1987, and only about one-tenth in 2012. A similar pattern holds for small firms. The number and employment share of entrepreneurial firms are in line with the typical high skewness in firm size and age distributions–much of the economic activity is concentrated in a relatively small fraction of firms in the right tail of these distributions. The average scale of young firms measured by employment is only about one-quarter of that for other firms, and much smaller for small firms.

Entrepreneurial firms also tend to have higher hiring and separation rates, and rely more on those individuals without jobs for filling vacancies.[17] As documented in Table 2, entrepreneurial firms, defined by age or size, accounted for about a quarter of quarterly gross hires from nonemployment and gross separations to nonemployment in 2000. Consider next the relative share of hires made by entrepreneurial firms that come from nonemployment, and the relative share of separations from entrepreneurial firms into nonemployment.[18] Both of these relative figures exceed one in Table 2, indicating that entrepreneurial firms disproportionately draw their workforce from the nonemployed, and lose their workers disproportionately to nonemployment, compared to other firms.

The differences in worker earnings, and hiring and separation patterns documented in Table 2 hint at potentially different labor market frictions for workers in entrepreneurial versus other firms. Moreover, the discrepancy in the employment share and average scale of these two types of firms can result, in part, from the more stringent financial and managerial constraints entrepreneurs face. The model in the next section studies how these factors influence the sorting of workers across firms and give rise to worker productivity, earnings, and wealth differentials across the two types of firms.

workers in Germany. Heyman (2007) also finds a similar pattern in Swedish data. More recently, Dinlersoz, Hyatt, and Nguyen (2013) provide evidence that new manufacturing establishments in the U.S. provide lower average earnings to their workers than older ones. Ouimet and Zarutskie (2014) also observe a similar gap in average earnings in the matched employer-employee data for the U.S.

[17] See Haltiwanger, Hyatt, and McEntarfer (2015) and Goetz et al. (2015).

[18] Specifically, define relative hires from nonemployment as $(H_e^N/H_e)/(H_c^N/H_c)$ where H_e^N is the total number of hires of entrepreneurial firms from nonemployment, H_e is the total number of hires by entrepreneurial firms, and H_c^N and H_c are the analogous measures for the non-entrepreneurial firms. Relative separations are defined analogously.

3 The Model

Based on the differences between entrepreneurial and other firms highlighted in the previous section, the model considers an economy with two sectors, entrepreneurial and corporate. The two sectors differ in production technologies, labor market frictions, and financial constraints. The model extends the framework of incomplete markets with occupational choice in the spirit of Quadrini (2000) and Cagetti and DiNardi (2006) to include heterogeneous labor markets, as in the "islands" economy of Lucas and Prescott (1974).[19] It also features indivisible labor choice characterized by frictions between production and leisure "islands", as in Krusell, Mukoyama, Rogerson, and Sahin (2011).

There is a unit mass of infinitely-lived individuals. Time, t, is discrete and the discount factor is $\beta \in (0,1)$. Each period an individual is endowed with one unit of time, which can be used for production as a worker or an entrepreneur. Individuals have identical preferences represented by the period utility

$$u(c_t, h_t) = \ln c_t - \alpha h_t,$$

where $c_t \geq 0$ is the consumption, $\alpha > 0$ is the disutility from labor, and $h_t \in \{0,1\}$ is an indicator of participation in the labor market as a worker or entrepreneur.

Each individual possesses an amount, $a_t \geq 0$, of assets.[20] Individuals also differ in their ability (or productivity), both as a worker and an entrepreneur. Worker productivity is summarized by $z_t > 0$–the efficiency units of labor an individual can supply in a period. The productivity, z_t, evolves over time independently across individuals according to the process

$$\begin{aligned} \ln z_t &= \rho_z \ln z_{t-1} + \epsilon_t^z, \\ \epsilon_t^z &\sim N(0, \sigma_z). \end{aligned} \tag{1}$$

Similar to the worker ability, the entrepreneurial ability, θ_t, also evolves independently across individuals according to

$$\begin{aligned} \ln \theta_t &= (1-\rho_\theta)\mu + \rho_\theta \ln \theta_{t-1} + \epsilon_t^\theta, \\ \epsilon_t^\theta &\sim N(0, \sigma_\theta). \end{aligned} \tag{2}$$

Production takes place in corporate and entrepreneurial sectors, denoted by $j \in \{f, e\}$, respectively. The two sectors have different production technologies. There is a representative firm in the corporate sector. It generates output, Y_t, by combining capital, K_t, and efficiency units of labor, L_t, through a constant-returns-to-scale production technology

$$Y_t = A K_t^\nu L_t^{1-\nu},$$

[19] See also Alvarez and Veracierto (2000).

[20] The index for an individual is suppressed for notational simplicity.

where $\nu \in (0,1)$, and $A > 0$ is the corporate sector's total factor productivity.

Each firm in the entrepreneurial sector is operated by an entrepreneur with ability θ_t, who uses capital, k_t, and efficiency units of labor, l_t, to produce output, y_t, via a decreasing-returns-to-scale technology

$$y_t = \theta_t (k_t^{\nu} l_t^{1-\nu})^{\xi}, \tag{3}$$

where $\xi \in (0,1)$ is a span-of-control parameter, which reflects the diminishing returns to the entrepreneur's managerial ability. Entrepreneurs also face a constant, exogenous probability of having their business purchased by the corporate sector. This feature captures, in a reduced form, the transition of firms from the entrepreneurial sector to the corporate sector.

There are two types of frictions. The first type are those in the labor market. Employment opportunities for nonemployed individuals arrive every period with probability λ. Job offers can come from the corporate sector or the entrepreneurial sector. Conditional on the arrival of a job offer, the offer is from the corporate sector with probability γ. Employed individuals maintain a deterministic match to their sector for the duration of their tenure. There is no on-the-job search, and individuals can receive job offers only when nonemployed. Every period workers can separate from their employers voluntarily or involuntarily. Involuntary separations occur in sector $j \in \{f, e\}$ with probability, ϕ_j. When an individual is separated from a firm or quits entrepreneurship, the individual has to stay nonemployed for at least one period before facing the decision to work or become an entrepreneur again. The parameters $\{\lambda, \gamma, \phi_f, \phi_e\}$ govern the frictions in the labor market.

The second type of frictions is financial in nature. There are borrowing constraints for entrepreneurs, and individuals are not allowed to carry negative assets, $a_t \geq 0$. The amount of capital, k_t, an entrepreneur with assets, a_t, can access is bounded: $k_t \leq b a_t$, where $b \geq 1$ is an exogenously given borrowing limit. When $b = 1$, entrepreneurs can only use their accumulated assets to finance production. The parameter b is the only parameter that governs the financial frictions for entrepreneurs. Capital rental rate is $r > 0$, and the depreciation rate is $\delta \in (0,1)$.

The timing of events within a period is as follows. Individuals first realize their current-period labor productivity, z. Each nonemployed individual then receives a job offer from one of the sectors. All individuals then make their decisions about whether to work, become an entrepreneur, or not work. Following this decision, all entrepreneurs realize their current-period abilities, θ, and choose their inputs for production. Each individual then chooses how much to consume and save. At the end of the period, some of the employed individuals get separated from their employers exogenously, and some entrepreneurs' businesses are bought out by the corporate sector with some probability, p. The entrepreneurs whose businesses transition into the corporate sector become nonemployed in the next period.

3.1 Individuals' Problems

Consider a stationary environment where policies and payoffs do not depend on calendar time. Let $s = (a, z, \theta)$ summarize an individual's assets, and worker and entrepreneurial ability in a period. In addition to s, each individual is differentiated by current-period labor status, which can be nonemployment (n), working in the corporate sector (f), working in the entrepreneurial sector (e), or being an entrepreneur (m). Similar to s, define $\widetilde{s} = (a, z, \theta_{-1})$ to be the individual's assets, worker, and entrepreneurial ability, before the current-period entrepreneurial ability, θ, is known. Note that $\widetilde{s}$ is identical to s except for its last element, which is the individual's previous-period entrepreneurial ability.

Consider now an individual who was a worker in sector j at the end of the previous period, or who has a job offer from sector j in the current period. This individual faces the choice between work in sector j, nonemployment (n), and entrepreneurship (m). The choice is made before the current period entrepreneurial ability is realized, but with the knowledge of current worker ability and assets. The expected value of this individual is then the maximum of the expected values from the three choices (j, n, m) available

$$E^j(\widetilde{s}) = \max\{\mathbb{E}_{\theta|\theta_{-1}}[V^j(s)], \mathbb{E}_{\theta|\theta_{-1}}[V^n(s)], \mathbb{E}_{\theta|\theta_{-1}}[V^m(s)]\}. \tag{4}$$

Consider next an individual who was not a worker in any sector at the end of the previous period, or who has no job offer in the current period. This individual faces the choice between nonemployment (n) and entrepreneurship (m), and his expected value is given by

$$U(\widetilde{s}) = \max\{\mathbb{E}_{\theta|\theta_{-1}}[V^n(s)], \mathbb{E}_{\theta|\theta_{-1}}[V^m(s)]\}. \tag{5}$$

Next, turn to the definitions of the value functions V^j, V^n, and V^m in (4) and (5). The value of a nonemployed individual can be written as

$$V^n(s) = \max_{c, a' \geq 0}\{\ln c + \beta \mathbb{E}_{z'|z}[\lambda[\gamma E^f(\widetilde{s}') + (1-\gamma)E^e(\widetilde{s}')] + (1-\lambda)U(\widetilde{s}')\}, \tag{6}$$

subject to the budget constraint

$$c + a' = (1 + r)a,$$

where $\widetilde{s}' = (a', z', \theta)$ and (a', z') denotes the next period's assets and worker ability. Equation (6) reflects the fact that a nonemployed individual obtains the utility from consumption in the current period, and in the next period the expected value depends on whether a job offer is received, and the sector this offer comes from.

Denote by w_j the wage per unit of worker efficiency in sector $j \in \{f, e\}$. The value of an individual who works in sector j is given by

$$V^j(s) = \max_{c, a' \geq 0}\{\ln c - \alpha + \beta \mathbb{E}_{z'|z}[(1-\phi_j)E^j(\widetilde{s}') + \phi_j U(\widetilde{s}')]\} \tag{7}$$

subject to

$$c + a' = w_j z + (1 + r)a,$$

The value in (7) is composed of two parts. An employed individual receives a current utility from consumption, reduced by the disutility of work. In the next period, the individual's expected value depends on whether he gets separated from his job.

Finally, the value of an entrepreneur is

$$V^m(s) = \max_{c,a'\geq 0}\{\ln c - \alpha + (1-p)\beta \mathbb{E}_{z'|z}[U(\tilde{s}')] + p(\tau(s) + \beta \mathbb{E}_{z'|z}[\mathbb{E}_{\theta'|\theta}[V^n(s)]])\} \tag{8}$$

subject to

$$c + a' = \pi(s) + (1 + r)a,$$

where the entrepreneurial profit, $\pi(s)$, is given by

$$\pi(s) = \max_{k,l\geq 0; k\leq ba}\{\theta(k^\nu l^{1-\nu})^{1-\xi} - w_e l - (r + \delta + \delta_\tau)k. \tag{9}$$

The entrepreneurial value in (8) consists of the current period utility that results from consumption and work, and the next period's expected value, which depends on whether the entrepreneur's business is bought out by the corporate sector. The individual can continue to be an entrepreneur or choose to be nonemployed in the event the corporate sector does not purchase the entrepreneur's firm, which occurs with probability $1-p$. If instead the firm is purchased (with probability p), the agent receives a transfer of $\tau(s)$ and becomes nonemployed in the following period.[21] The transfer, $\tau(s_t)$, is equal to the present discounted value of future stream of profits of the entrepreneur starting from the period of transfer t

$$\tau(s_t) = \pi(s_t) + \sum_{k=1}^{\infty}\left(\frac{1}{1+r}\right)^k \mathbb{E}_{\theta_{t+k}|\theta_{t+k-1}}[\pi(s_{t+k})]. \tag{10}$$

In other words, the corporate sector pays a transfer to an entrepreneur with state s that equals the expected value from perpetually operating the entrepreneurial business starting from the period business is purchased. In equilibrium, the total amount of transfers from the corporate sector to the entrepreneurs is financed by a portion, δ_τ, of the capital used by the corporate sector, K.[22]

[21] The fact that p is exogenous and does not depend on entrepreneurial ability is a crude way of formulating the transition of an entrepreneurial firm to the corporate sector. One could think of a more elaborate formulation where p is a function of the state s. However, the goal here is not to provide a model of transition over the life-cycle of an entrepreneur. In practice, the way this transition is modelled makes little difference in the framework here as long as such transitions are rare events – at least, the low frequency of IPOs and M&A activity in the U.S. economy supports this view, as discussed in the model's calibration.

[22] Note that in the value of the business on the right hand side of (10) the discount rate is $1/(1 + r)$, rather than the subjective discount rate, β, of individuals. The former rate is the relevant one here since the corporate sector uses capital to purchase an entrepreneurial business, and the capital rental rate is r.

3.2 Equilibrium

Let $i \in \{n, f, e, m\}$ denote the labor status of an individual in any given period. In addition, let $d \in \{n, f, e, m\}$ be the "island" or "location" of the individual at the end of the previous period. A stationary competitive equilibrium for the model is a collection of value functions, $V^i(s)$, wage in each sector, w_j for $j \in \{f, e\}$, an interest rate, r, a transfer rate, δ_τ, labor supply rules, $h^d(\widetilde{s})$, decision rules to become an entrepreneur, $m^d(\widetilde{s})$, saving and consumption rules, $a'^i(s)$ and $c^i(s)$, an entrepreneur's capital and labor choice rules, $k(s)$ and $l(s)$, and measures of individuals by labor status, $\Psi^i(s)$, such that

1. The labor supply rules, $h^d(\widetilde{s})$, and the decision rules to become an entrepreneur, $m^d(\widetilde{s})$, solve the problems (4) and (5),

2. The saving and consumption rules, $a'^i(s)$ and $c^i(s)$, solve the individuals' problems defined in (6), (7), and (8),

3. The interest rate, r, and the corporate sector wage, w_f, satisfy

$$r = \nu A K^{\nu-1} L^{1-\nu} - \delta - \delta_\tau, \tag{11}$$

$$w_f = (1-\nu) A K^{\nu} L^{-\nu}, \tag{12}$$

4. The transfer rate, δ_τ, ensures that total amount of transfers to entrepreneurs are accounted for by a portion of the corporate sector capital

$$\int p\tau(s) d\Psi^e(s) = \delta_\tau K, \tag{13}$$

5. The capital and labor choices, $k(s)$ and $l(s)$, solve the entrepreneur's problem in (9),

6. The measures, $\Psi^i(s)$, are consistent with the transitions of the individuals across islands,

7. Labor, capital, and goods markets clear

$$\int l(s) d\Psi^m(s) = \int z d\Psi^e(s) \qquad \text{(entrepreneurial sector labor)} \tag{14}$$

$$L = \int z d\Psi^f(s) \qquad \text{(corporate sector labor)} \tag{15}$$

$$K + \int k(s) d\Psi^m(s) = \sum_i \int a d\Psi^i(s) \qquad \text{(capital)} \tag{16}$$

$$Y + \int y(s) d\Psi^m(s) = \sum_i \int c(s) d\Psi^i(s) + \delta_\tau K + \delta \left(K + \int k(s) d\Psi^m(s) \right) \qquad \text{(goods)} \tag{17}$$

where $y(s)$ denotes the output of an entrepreneur with state s.

While the corporate sector wage, w_f, depends on the representative corporate firm's labor choice (12), the entrepreneurial sector wage, w_e, is the value that equates the labor demand by all entrepreneurs to the labor supply of all workers in the entrepreneurial sector–equation (14). The amount of capital used by the two sectors must equal the total assets of all individuals in the economy, as ensured by (16). Finally, the total output of the economy must account for the total consumption by individuals, the replacement of the depreciated capital, and the transfers to entrepreneurs from the corporate sector, as shown in (17). Appendix A outlines the algorithm that is used to solve for the stationary equilibrium numerically.

4 Calibration

The parameter values used in the calibration of the baseline model are shown in Table 3. Each period corresponds to one quarter. The discount rate, β, is set to 0.98, to match an annual interest rate of 4%. The process for labor productivity, z, in (1) is assigned the quarterly versions of annual parameters estimated by Heathcoate, Storesletten, and Violante (2010). The annual parameters are $\{\rho_z, \sigma_z\} = \{0.97, 0.13\}$.[23] Similar persistence and standard deviation values are obtained when, instead, a quarterly transition process is estimated using quarterly earnings data from Longitudinal Employer-Household (LEHD) data directly. The details of this estimation are available in Appendix A.

The annual values of the parameters $\{\rho_\theta, \sigma_\theta\}$ of the process for managerial ability θ in (2) and the returns-to-scale parameter, ξ, are estimated separately for entrepreneurial firms (firms aged 0-5 years) versus non-entrepreneurial firms (firms aged 6+ years) in the manufacturing sector. The unavailability of data on inputs other than labor precludes the estimation of these parameters for firms in other sectors of the economy. The estimation follows the econometric methodology used in Abraham and White (2015), which allows joint estimation of the parameters $\{\rho_\theta, \sigma_\theta, \xi\}$ based on Castiglionasi and Ornaghi (2013) – see Appendix B.[24] The framework of Abraham and White (2015) has a number of desirable features. Notably, it allows for heterogeneity in the parameters $\{\rho_\theta, \sigma_\theta, \xi\}$ across industries. They demonstrate that restricting these parameters to be the same across industries can lead to significant upward bias in the estimate of the persistence parameter, ρ_θ.[25] The estimated parameters for the entrepreneurial ability process for θ at an annual rate

[23] The support of the labor productivity process is discretized using 21 grid points based on the Rouwenhorst method. See Kopecky and Suen (2010).

[24] See also the earlier version, Abraham and White (2006).

[25] Other recent approaches to estimating the productivity persistence parameter include Lee and Mukoyama (2015) and Foster, Haltiwanger and Syverson (2008). The former does not allow for heterogeneity of the persistence parameter, and the latter provides estimates for a sample of 11 narrowly defined manufacturing product categories. These approaches generally result in higher estimates for the persistence parameter.

turn out to be $\{\rho_\theta, \sigma_\theta\} = \{0.3, 0.18\}$, which are the averages across narrowly defined industries at the level of 4-digit SIC codes. The span-of-control parameter for young firms, ξ, has an average estimated value of 0.88 across industries. This value is smaller than the corresponding one for older firms (around 0.97), suggesting a lower span-of-control for young firms.

Following Kitao (2008) and Buera and Shin (2011), the borrowing constraint parameter, b, is set to 1.5, implying that an entrepreneur can borrow up to 50% of his assets at the beginning of the period. Based on the business-cycle literature, the capital's share of output, ν, is set to 0.36, and the quarterly depreciation rate, δ, is taken to be 0.015, which corresponds to an annual depreciation rate of 0.06. The productivity of the corporate sector, A, is normalized to $\exp(-1)$.

In the model, entrepreneurs face a constant probability, p, of transitioning into the corporate sector. One can think of this transition as an IPO, a purchase of the entrepreneurial business by a corporate firm, or an exceptionally high firm growth that removes the constraints the entrepreneur faces. Consider the case of an IPO. Based on Dun and Bradstreet's Compustat database, the number of publicly-traded firms in any given year during the period 2002-2012 is around 5-6 thousand. This set of firms represent about 0.1% of the entire set of employer businesses in the U.S. during the same period, which range around 5-6 million. Similarly, the number of announced mergers and acquisitions in the U.S. ranged approximately from 8 to 14 thousand during the period 2000-2015, indicating that at most a fraction of about 0.1%-0.2% of firms engage in this type of announced merger and acquisition activity.[26] Given these estimates, a value of $p = 0.001$ is chosen to approximate the flow of entrepreneurial firms into the corporate sector. The analysis is robust to different choices of p. For instance, a value of $p = 0$ on the low end, or $p = 0.002$ on the high end, produce very similar results.

The remaining parameters, denoted by the vector $M = \{\alpha, \lambda, \gamma, \phi_e, \phi_f, \mu\}$, are chosen to hit six different targets that constitute a system of non-linear equations. While these equations are simultaneous in nature and involve all relevant parameters of the model, each equation plays an instrumental role in setting a specific parameter. The values of the targets are chosen to be the average value of their empirical counterparts in the early 2000s. For the disutility of labor, α, the key target is the employment-to-population ratio (0.80) among males aged 15-64 years. Two other targets, the share of employment in non-entrepreneurial firms (84%) and the average worker earnings premium for these firms (17%), are important in pinning down a value for the job offer rate from the corporate sector, γ, and for the separation probability from entrepreneurial sector employment, ϕ_e. The job finding rate, λ, and the job separation rate from corporate sector employment, ϕ_f, are set so that the aggregate job separation rate (employment-to-nonemployment flows) is 1.9% as a fraction of total employment, and the aggregate job finding

[26] These figures are provided by Institute of Mergers and Acquisitions, available at https://imaa-institute.org/m-and-a-us-united-states/.

rate (nonemployment-to-employment flows) is 45%, based on Shimer (2012). Finally, the fraction of entrepreneurs, 4%, is targeted in assigning a value to the entrepreneurial ability parameter, μ.The fraction, 4%, is obtained from the compilation of estimates based on a wide-range of definitions and sources in Table 1. See Appendix A for details of the calibration procedure.

The calibration procedure yields an overall job offer rate of $\lambda = 0.52$, and a (conditional) corporate sector job offer rate of $\gamma = 0.92$. The (unconditional) entrepreneurial sector job offer is then $\lambda(1-\gamma) = 0.042$, which implies significant frictions that impede worker flows into the entrepreneurial sector. Furthermore, separation rate is higher in the corporate sector: $\phi_f = 0.012 > \phi_e = 0.000$. While nonemployed individuals have a lower likelihood of getting an entrepreneurial job offer, entrepreneurial jobs also have very low exogenous separation rates; in fact, ϕ_e is very close to zero. This low separation rate, however, should not be interpreted as a worker's tenure in any given entrepreneurial firm being necessarily longer than that in a corporate firm. In the model's framework, the low separation rate instead means that, once employed, a worker sojourns much longer on average in the entrepreneurial sector than in the corporate sector, and leaves the sector almost always voluntarily. Since the firm a worker matches with in the entrepreneurial sector is not identified, this longer expected stay in the entrepreneurial island may be thought of frictionless switching from job to job *within* the entrepreneurial sector, although such job dynamics is not explicitly modelled.

5 Properties of the Baseline Model

The key features of the calibrated model's equilibrium are shown in Table 4. Overall, the model comes close to matching the targeted values. It produces an employment-to-population ratio of 82%, and around 5% of the individuals choose to become entrepreneurs. As shown in Figure 1a, individuals with a higher level of entrepreneurial ability tend to become entrepreneurs – the distribution of managerial ability for entrepreneurs stochastically dominates that for non-entrepreneurs, in a first order stochastic sense. Entrepreneurs also tend to have higher levels of assets (Figure 1b). Furthermore, entrepreneurial firms exhibit variation in their capital input, which has a skewed distribution (Figure 1c). The distribution of the labor input (in efficiency units) for the entrepreneurial firms in Figure 1d is also highly-skewed.[27] The features of the model discussed so far also emerge in recent models of entrepreneurship, indicating that the model is able to capture the salient aspects of these models.[28]

[27] This shape is in line with the typical shape of the firm-level distributions of labor input in empirical studies. However, note that the labor input in the model (worker efficiency units) is different from the employment measure (the number of workers) typically used in empirical studies of firm size.

[28] See, e.g., Quadrini (2000), Cagetti and De Nardi (2006), Kitao (2008), Buera and Shin (2011), Buera, Fattal-Jaef, and Shin (2015), and Bassetto, Cagetti, and De Nardi (2015).

The model's main distinguishing aspect, heterogeneous labor markets, provides further insight to the functioning of the labor markets and the nature of worker sorting. The calibrated model's equilibrium generates patterns that are broadly consistent with the behavior of the key metrics for the U.S. labor market. In the baseline model, 15% of the employees work for young firms, close to the data counterpart of 16%, as seen in Table 4. The model also delivers a corporate earnings premium consistent with its observed value. The average worker earnings in the corporate sector is about 19% higher than that in the entrepreneurial sector, and is close to its targeted value of 17%. Note that the sectoral average worker earnings depends on the distribution of worker productivity (z) in each sector, as well as the wages per efficiency units of labor (w_e, w_f). The values for w_e and w_f are obtained in the calibrated model's equilibrium, but there is no observable target to discipline their values. The wage per efficiency unit of labor in the corporate sector turns out to be 2% higher than that in the entrepreneurial sector. Moreover, a worker in the corporate sector is 17% more productive, on average, than a worker in the corporate sector. This sorting of individuals based on productivity drives in part the corporate earnings premium, as explored in more detail later. Note also that the fraction of corporate capital transferred from the corporate sector to the entrepreneurs is estimated to be $\delta_\tau = 0.00003$ (transfer rate). This small fraction ensures that about 0.1% ($p = 0.001$) of entrepreneurs are bought out by the corporate sector every period.

Figure 2a shows that individuals with higher managerial ability tend to become entrepreneurs. As managerial ability increases, individuals tend to shift from corporate sector employment to entrepreneurship, with little change in the allocation of individuals into entrepreneurial sector work across managerial ability levels. Figure 2b illustrates how individuals at a given worker productivity level are allocated across the two sectors and entrepreneurship. As worker productivity increases, the fraction of individuals who work in the corporate sector increases, whereas the fraction of individuals who are entrepreneurs declines. However, the fraction employed in the entrepreneurial sector is not monotonic. Apart from the spikes at the lowest and highest levels of labor productivity, the fraction of workers in the entrepreneurial sector tends to decrease as labor productivity increases. As discussed below, this non-monotonicity is driven by how the distribution of assets across workers influences their decision to work.

Figure 2c shows the distribution of workers' assets by sector. The distribution in the entrepreneurial sector is much more skewed, with a high mass over the range of low asset levels. Table 4 indicates that the average assets of the workers in the corporate sectors is about 2.3 times that in the entrepreneurial sector. When only the workers in their first quarter of a job is considered, the average assets for workers in the corporate sector is about 1.3 times that of those in the entrepreneurial sector – see Figure 2d for the distributions of assets by sector for these workers. That is, there is an average wealth differential not only between the workers in the two sectors, but also between the workers who have just *accepted* jobs (those in their first period of their employment spell) in these two sectors. The asset ratios in the model broadly line up with the direction of sorting observed in the data, even though asset ratios are not explicitly targeted in the calibration of the model–see Section 7 for further empirical analysis.

If entrepreneurial firms pay lower wages per efficiency unit, why does anyone work for them at all? The answer lies in the patterns exhibited by the average assets ratios in the two sectors. Because nonemployed individuals with low assets are not wealthy enough to secure a smooth stream of consumption while unemployed, they cannot afford to reject a job offer from the entrepreneurial sector and wait for a job offer from the corporate sector. In other words, the opportunity cost of waiting for a corporate offer is high for these individuals. Therefore, they more readily take entrepreneurial job offers. This sorting of workers also has implications on earnings and wealth inequality. On the one hand, the presence of the entrepreneurial sector allows nonemployed, low-wealth individuals to build assets. On the other hand, the wage and earnings differentials between the two sectors leads to a faster accumulation of wealth for workers in the corporate sector, who are more wealthy to start with. While the former effect works to reduce wealth inequality, the latter can propagate inequality.

Figure 3a shows what types of individuals accept a job offer from the entrepreneurial versus corporate sector. The figure illustrates the acceptance/rejection regions for entrepreneurial and corporate offers by worker productivity and assets for the median managerial ability. As an individual's assets increase, the threshold productivity for accepting a corporate sector job offer increases. Note also that, given assets, the productivity threshold for accepting a corporate sector offer always lies below the one for an entrepreneurial sector offer. In other words, individuals who take jobs in the entrepreneurial sector tend to be more productive. The wage differential implies a higher return to work in the corporate sector and generates an incentive for individuals to wait for a corporate job offer. However, the higher job separation rate in the corporate sector suggests that the return to work in corporate sector cannot be too large.

A notable feature of Figure 3a is that for an entrepreneurial job offer there is a portion of the rejection region that protrudes into the acceptance region, labeled as Region B. The presence of Region B indicates a non-monotonicity in the decision rule to accept employment in the entrepreneurial sector. Note that no such region exists for the decision rule for corporate sector employment. Figure 3b shows the acceptance region for becoming an entrepreneur at the median managerial ability. This figure indicates that individuals would not choose to become entrepreneurs inside Region B highlighted in Figure 3a. In other words, individuals with types in Region B reject a job offer from the entrepreneurial sector in favor of continuing to be nonemployed and waiting for a corporate sector job offer.

One way to further understand the nature of Region B in Figure 3a is to examine the value functions for an individual with median entrepreneurial ability and median labor productivity, plotted in Figure 3c. The figure plots value functions for some subset of the parameter values underlying Region B. The value of leisure exceeds that of entrepreneurial work in that region, and both value functions are well-behaved. To understand the source of Region B further, consider the entrepreneurship choice in a partial equilibrium setting. Holding prices (w_f, w_e, r) fixed at their baseline values, suppose entrepreneurship is no longer available as a choice. What does the optimal decision rule look like for an individual with an entrepreneurial sector job offer? Figure 3d shows this new decision rule for an individual with median managerial ability. In this partial-equilibrium setting, individuals are much more choosy about accepting a job in the entrepreneurial sector. The main reason is that an incentive to work in order to accumulate assets to finance a potential entrepreneurial project in the future is now absent. In other words, there is no incentive to accumulate capital outside the precautionary savings motive. As a result, the threshold productivity above which individuals would choose to work is higher than in the baseline economy.

Now consider again the baseline economy with a choice of entrepreneurship, but without any uncertainty in managerial ability at the time the entrepreneurship choice is made. That is, suppose that the timing of events is such that θ and z are both realized at the beginning of the period. The value from becoming an entrepreneur is now known before the entrepreneurship decision is made. The dashed line in Figure 3d represents the acceptance threshold in such an economy, with prices fixed again at their baseline values. In this economy individuals have a lower threshold for accepting employment opportunities, compared with the baseline economy. In particular, a region like Region B in Figure 3a is not present in Figure 3d. An individual with a realization of assets and productivity in Region B rejects a job in the entrepreneurial sector in the baseline economy, but accepts such a job in the economy without uncertainty about the managerial ability. The reason is that the uncertainty about the optimal scale of the entrepreneurial firm reduces the ex-ante return to working to accumulate assets to potentially become an entrepreneur in the future. This region disappears for low-asset individuals whose return from entrepreneurial activity is smaller, and for high-asset individuals who can operate firms at their optimal scale even under the uncertainty.

The model also embeds a few mechanisms that generate earnings and wealth inequality. While high-wage corporate sector employment and entrepreneurship options can help individuals accumulate wealth at a higher rate, episodes of low-wage entrepreneurial sector employment and nonemployment can slow down such accumulation. The availability of jobs in the entrepreneurial sector nevertheless allows individuals who would otherwise be nonemployed to generate wealth, compared to an environment where there is no entrepreneurial sector. Additionally, the option to become an entrepreneur itself enables some individuals to reach the left tail of the wealth distribution. Note that the wealth or earnings inequality moments are not targeted as part of the baseline calibration. However, one can get a sense of how well the model performs in generating earning and wealth inequality by comparing the shares of wealth and earnings in different quintiles of the respective distributions to those obtained from the Panel Study of Income Dynamics (PSID) in Table 5 (see An, Chang, and Kim (2009)). The table indicates that the model does fairly well in capturing the observed distribution of earnings and wealth, though less so for the top quintile's share of total wealth in the economy – 65% in the model versus 76% in the data. The model's performance regarding inequality is comparable to that of more standard models of a heterogenous agent economy (e.g., An, Chang, and Kim (2009)) than the models of entrepreneurship such as Cagetti and Dinardi (2006) or Buera and Shin (2011), where calibration explicitly targets the moments of the wealth distribution.

6 An Analysis of the Model's Key Features

This section explores how some key features of the model drive the nature of worker sorting and equilibrium allocations across the two sectors. The analysis focuses on the roles of labor prices, uncertainty about entrepreneurial ability, borrowing constraint, and entrepreneurial sector job offer rate.

6.1 Labor Prices

The model is distinguished by the presence of two different prices for labor, unlike most models of entrepreneurship which feature a uniform price. A key question is then to what extent the presence of different sectoral wages drive worker sorting and equilibrium allocations. To answer this question, two approaches are considered. In the first approach, the value of the entrepreneurial wage is forced to be equal to the value of the baseline model's corporate sector wage. The remaining prices and all other model parameters, including search frictions, are held at their baseline values. This approach explores the effects of raising the entrepreneurial sector wage to the corporate level without allowing the rest of the model parameters to respond to this change. The second approach is to let the corporate sector set a uniform wage for the entire economy, and to do the calibration exercise again to recover a new set of values for the parameters, M, that match the data targets under this uniform price – instead of leaving the parameters fixed at their baseline values as in the first exercise.[29] The second approach therefore allows the model's parameters adjust to reach a new equilibrium under the uniform wage .

[29] One drawback of this approach is that the wage set by the corporate sector does not necessarily clear the labor market for the entrepreneurial sector.

Table 6 compares the results of the first approach with the baseline. When the entrepreneurial sector wage is raised to match the corporate sector wage, allocations change. The higher wage in the entrepreneurial sector substantially increases incentives for individuals to work in this sector. As a result, the entrepreneurial sector employment share increases from 15% in the baseline to 33%. This sharp increase illustrates that relatively low wage in the entrepreneurial sector in the baseline model plays a significant role in reducing the incentives for workers to take jobs in this sector. Note also that when the entrepreneurial sector wage is as high as in the corporate sector, the entrepreneurship rate declines from 5% to 3%, as entrepreneurs now have higher labor costs. More importantly, worker sorting prevails, even though the degree of sorting is not as high as in the baseline. The question is then what else drives worker sorting, if not the wage differential alone. Recall that the corporate separation rate is higher than the entrepreneurial separation rate ($\phi_f > \phi_e$). Now, suppose ϕ_e is forced to equal ϕ_f, in addition to the equal wages. As shown in the third column of Table 6, the ratios of worker productivity and average assets are now all unity. That is, worker sorting vanishes, suggesting that differential separation rates are important in inducing worker sorting in the absence of a wage differential. When workers face a shorter expected tenure in the corporate sector and the wages are equal across the two sectors, only the relatively more productive and wealthy workers sort into the corporate sector.

Table 7 summarizes the results of the second approach. While most of the model's parameters change to some extent with respect to the baseline, the most notable change is in the corporate sector job offer rate, γ, which rises from 0.92 to 0.96. This increase is needed to counter the higher flow of workers into the entrepreneurial sector which offers a wage as high as that in the corporate sector, and to ensure that the targeted entrepreneurial sector employment share is achieved. In other words, imposing a uniform price for labor requires larger job finding frictions in the entrepreneurial sector to limit the employment share in that sector. The slightly lower separation rate in the corporate sector also helps achieve this target by making the corporate sector more attractive for workers than in the baseline. Finally, note that the average entrepreneurial ability increases from its baseline value. Entrepreneurs now have to be more productive on average to be able to afford the higher wages they face.

6.2 Uncertainty about Entrepreneurial Ability

In the baseline model, the timing of the resolution of the entrepreneurial uncertainty is such that the entrepreneurial ability draw is realized *after* the decision to become an entrepreneur is made. How much does this uncertainty matters for worker sorting and equilibrium allocations? Table 8 compares key features of the baseline economy with an alternative economy where individuals *know* their initial entrepreneurial ability draw at the time of the entrepreneurship decision. While the economy without uncertainty replicates the overall employment rate, the employment shares by sector, and the entrepreneurship rate, there is a notable difference in the sectoral wages compared to the baseline model. In particular, in the economy with no uncertainty w_e exceeds w_f, in a reversal of the baseline ranking. This discrepancy reflects the changes in the types of individuals who become entrepreneurs. The absence of uncertainty leads to larger entrepreneurial businesses, on average, relative to the baseline economy. This effect is seen in Figure 4. The entrepreneurial ability distribution in the economy with no uncertainty places more mass on larger entrepreneurial projects (higher θ) compared to the baseline economy. As a result, while 90% of the entrepreneurs are borrowing constrained in the baseline economy, virtually all of the entrepreneurs are constrained in the economy with no uncertainty. Because entrepreneurs now have larger and more profitable businesses, the entrepreneurial sector can support a higher wage than the corporate sector.

How does the model with no uncertainty still generate a corporate sector earnings premium of 17%, despite the fact that the corporate sector wage is now lower? The parameters M recovered for the economy with no uncertainty are shown in Table 8. While there is some change in all parameter values relative to the baseline model, the probability of a corporate job offer notably increases from 0.92 to 0.97. This increase makes jobs in the entrepreneurial sector scarce and restricts the flow of workers into that sector, despite the higher wage in that sector. Consequently, the model is still able to capture the relatively small employment share of the entrepreneurial sector. In addition, because the corporate sector now offers a lower wage, it becomes less attractive for workers. This effect induces only the relatively more productive workers to sort into that sector – 19% more productive on average than the workers in the entrepreneurial sector, compared to 17% in the baseline economy. As a result, a corporate earnings premium close to the one in the baseline still emerges.

6.3 Borrowing Constraint

The baseline model sets the borrowing limit at 50% of an entrepreneur's assets ($b = 1.5$). This value of b is common in earlier models of entrepreneurship (see, e.g., Kitao (2008) and Buera and Shin (2011)), but is certainly not the only way to specify the extent of limits to borrowing. To assess the role of the borrowing constraint further, the borrowing constraint parameter b is increased to 2.0. The parameters, M, are again recovered to match the targets in the baseline with the new borrowing constraint in place.

Table 9 compares the equilibrium allocations, prices, and parameters for the baseline economy to that for the economy with $b = 2.0$. The corporate sector job offer rate, γ, in the case of $b = 2.0$ is higher than that in the baseline economy. The ratio of wages is similar to that found in the case with no uncertainty for entrepreneurial ability. Once again, w_e exceeds w_f, as entrepreneurs now operate larger and more profitable businesses on average. In order to match the sectoral employment shares and the corporate sector earnings premium, the entrepreneurial job offer rate $(1-\gamma)$ decreases to generate higher job finding frictions. While a higher borrowing limit encourages entrepreneurs to expand, a lower average entrepreneurial ability compared to the baseline case works to limit the employment in the entrepreneurial sector. Again, as in the case of no entrepreneurial uncertainty, the corporate earnings premium of 15% emerges, despite the higher wages in the entrepreneurial sector. Worker sorting by productivity and assets prevail under $b = 2.0$, although it is less pronounced.

6.4 Entrepreneurial Sector Job Offer Rate

In the baseline model, the conditional probability that a job offer is from the corporate sector is $\gamma = 0.92$, which implies that most of the nonemployed individuals face offers from the corporate sector (conditional on receiving an offer). The previous sections highlighted the role of γ in the model's ability to match the key targets for the entrepreneurial sector. Here, the role of γ is examined in more detail. The equilibrium of the model is computed for values of γ in the set $\{0.1, 0.2, 0.4, 0.5, 0.6, 0.8, 0.92, 0.97\}$, holding all other parameters fixed at their baseline values. A key question is whether an exogenous reduction in job finding frictions in the entrepreneurial sector leads to an increase in entrepreneurship.

A lower γ (higher job finding rate in the entrepreneurial sector) has a positive effect on entrepreneurship (Figure 5a). The rate of entrepreneurship increases to almost 15% as γ approaches 0.1. This large increase over the baseline economy's entrepreneurship rate (5%) demonstrates the critical role of labor market frictions in generating incentives for entrepreneurship. The baseline value of $\gamma = 0.92$ is important in the model's ability to match the observed entrepreneurship rate.

As entrepreneurial sector hiring frictions decline (as γ decreases), the share of employment in the entrepreneurial sector also increases. In addition, the corporate earnings premium generally increases, along with the wages in the entrepreneurial sector (Figure 5b). When entrepreneurial sector jobs are harder to find (higher values of γ), this sector has to offer a higher wage per efficiency unit to attract workers, which also results in higher relative average worker earnings in the entrepreneurial sector.

Note that, despite the reduction in the sectoral wage gap as γ increases, workers are not any more evenly distributed across the two sectors based on average worker productivity and assets, as shown in Figure 5c. In fact, for lower values of γ worker sorting by assets and productivity appears to become *more* pronounced. The intuition for this result lies in the role of differential separation rates. Recall that the corporate sector has significantly larger exogenous separation rates than the entrepreneurial sector. As γ decreases and entrepreneurial sector jobs become easier to find, there are higher incentives to accumulate assets among workers in the corporate sector to insure against the relatively large probability of job loss. In addition, the incentives to sort into a corporate sector job increases for those individuals with higher assets, as indicated by the asset ratio of individuals in transition to employment.

Figure 5d provides one measure of selection into entrepreneurship as γ changes. Because an increase in γ is associated with an increasingly higher wage in the entrepreneurial sector, the cost of entrepreneurship goes up. This higher cost implies that only those individuals with relatively higher entrepreneurial ability are able to start businesses. In particular, the average entrepreneurial ability increases monotonically as γ moves from 0.1 to 0.97.

7 Evidence on Worker Sorting by Assets

A key prediction of the baseline model is that the average asset holdings of workers in the entrepreneurial sector is lower than that of workers in the corporate sector. This difference emerges as a result of two effects. First, workers holding fewer assets tend to accept job offers from the entrepreneurial sector. Second, workers in the corporate sector tend to accumulate assets at a higher rate because of higher wages and average earnings in that sector. Is there empirical evidence on this type of worker sorting based on assets? The answer requires data on workers' assets and the characteristics of their employers.

7.1 Data

Unfortunately, household survey data that include information on worker assets typically do not contain information on the age or size of a worker's employer. Towards addressing this shortcoming, this paper brings together two sets of data. The wealth data for workers in the Survey of Income and Program Participation (SIPP), which contains information on various assets held by individuals, are merged with the Longitudinal Employer-Household Dynamics (LEHD) data that captures employment spells, earnings, and some employer characteristics for those workers.

To measure worker assets, the responses in the Asset and Liabilities Topical Module collected in the 1996, 2001, 2004 and 2008 SIPP panels are used to create a net worth variable, excluding housing equity. The net worth variable is calculated at the household level, and used as the primary empirical counterpart to assets in the model. The net worth is the sum of the market value of assets owned by every member of the household (except housing) minus the liabilities of household members.[30] For the analysis, the net worth variable is winsorized at the top 1% to reduce the effects of some likely outliers. The SIPP data also provide a set of variables that describe individuals' characteristics, including gender, race, marital status, education, and age.

The individuals in the SIPP panels are linked to the LEHD data based on unique time-invariant individual identifiers called Protected Identification Keys (PIKs). The LEHD data are the universe of quarterly wage data for available states, and the SIPP quarter is the calendar quarter in which an individual's response to the survey is recorded. Because different states enter LEHD data at different times, the sample is restricted to data starting one year after the inclusion of a state's data series in the LEHD. This approach reduces spurious identification of new hires. The LEHD data also provide age and size measures for the firms individuals work for. In addition, industry affiliation for these firms are also available in the data. For workers holding more than one job during the relevant quarter, firm age and size pertain to the firm where worker earnings were the greatest among all jobs held in that quarter.[31] In the analysis to follow, all variables denominated in dollars are converted to 2014 constant dollars, and survey weights in SIPP are used to obtain estimates representative of the population.

7.2 Results

The top panel of Table 10 shows the mean and quasi-median of net worth. To be consistent with the calibrated model, the sample is restricted to male workers aged 15-64 in young (0-5 years of age) versus old firms (5+ years of age).[32] All statistics indicate a stark difference in average asset holdings of workers at young firms relative to old firms. In particular, workers in old firms have a mean net worth that is about 17% percent higher than that of workers in young firms. Based on the quasi-median, this difference is even larger – about 85%.[33] Note, however, that higher wages in established firms relative to young firms would imply an asset differential even when workers accept job offers randomly regardless of their asset holdings, as

[30] The assets included in this definition are interest-earning assets held at financial institutions, stocks and mutual fund shares, rental property value and rental income, home value, IRA, 401K and thrift savings plans, and values of vehicles owned. The excluded assets are equities in pension plans, the cash value of life insurance policies, and the value of home furnishings and jewelry.

[31] For details on how separate jobs and employment spells are identified in the LEHD data, see Hahn, Hyatt, and Janicki (2016).

[32] Quasi-median is calculated as an alternative to percentiles to satisfy U.S. Census Bureau's requirements for disclosure. It is the average of the observations in the data between the 45th and the 55th percentiles. The standard error for the quasi-median is obtained using a bootstrap procedure that calculates the quasi-median on 100 different 50% samples taken from the data.

[33] In addition, the gap between the net worth of workers generally persists at the first and third quartiles, indicating that worker sorting prevails at different parts of the asset distribution. The results for these quartiles are available upon request.

long as employment has some persistence. On this point, Table 10 shows that the earnings of workers in older firms are in general much higher, regardless of whether the earnings is measured by the mean or quasi-median.

Further evidence on sorting of workers based on assets at the time when they take jobs can be provided by examining the assets of recently hired workers only. For this purpose, the sample is restricted to those workers who are within their first year of employment following an unemployment spell. This subsample allows for an approximation to the asset holdings of workers in the model who have recently transitioned into employment from unemployment. The results by firm age are shown in the top panel of Table 10. There is an asset differential across workers in the two types of firms when measured by either the mean or quasi-median net worth. Therefore, the asset gap is not merely a result of the fact that working for older, more established firms allows individuals to accumulate more assets over time. Workers accepting jobs in these firms are on average wealthier to start with, in line with the model's prediction on sorting based on assets.

The bottom panel of Table 10 shows the extent to which worker sorting prevails when firm size is used instead of firm age to define entrepreneurial firms. For this purpose, a size threshold of 50 employees is used to define small versus large firms. Although the results are generally weaker compared to the results for firm age, the net worth differential remains. The median worker in large firms has net worth of about \$25,000 compared with nearly \$15,000 in small firms. The difference is smaller for recently hired workers. Similar to the case of firm age, earnings differentials are also present in the case of firm size. For larger firms, the mean earnings are much higher for all workers and recently hired ones.

One obvious question is how much of the observed worker sorting by assets is driven by other factors, such as worker characteristics. For instance, an individual's assets are likely correlated with marital status, education, and experience. To further isolate the connection between the type of the firm an individual works for and the worker's assets, Table 11 presents several estimates based on linear models of the form

$$\widetilde{a}_{it} = \sum_k \alpha_k d_{ikt} + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}, \tag{18}$$

where $\widetilde{a}_{it}$ is the inverse hyperbolic sine transformation of the net worth a_{it} for individual i in year t

$$\widetilde{a}_{it} = \ln(a_{it} + \sqrt{a_{it}^2 + 1}). \tag{19}$$

The transformation in (19) is used for at least two reasons.[34] First, the net worth variable a_{it} has a highly-skewed, non-normal distribution. Second, there are zero and negative values for

[34] See, for instance, Friedline, Masa and Chowa (2015) for an exploration of the properties of this transformation in the case of wealth data. Other transformations of the net worth variable were also used for robustness checks.

a_{it}, which leads to omitted observations when the standard log transformation is used. On the right hand side of specification (18), d_{ikt} is a dummy variable for the age or size category k the firm the individual works for is classified in, $\mathbf{x}_{it}$ is a vector of controls, and ε_{it} is an error term. The controls $\mathbf{x}_{it}$ include the individual's gender, race, marital status, education level, age, and age-squared, as well as industry and year fixed effects. The model in (18) is estimated using OLS for mean effects. In addition, median regression is used to assess the effects at the median of the net worth distribution. The primary coefficients of interest are α_k's, which measure the connection between net worth and firm age (or size). The regressions are run separately with firm age and firm size categories.

Table 11 indicates that both the age and size categories are significantly associated with net worth, even after controlling for a large set of observables. The estimates for firm age and size categories suggest that, compared to the omitted category of older firms (11+ years of age) or larger firms (500+ employees), workers in younger or smaller firms tend to have much lower net worth. This result holds for both the mean and median regressions, and for all young firm categories considered in the regressions. The results are generally stronger for all workers compared to recent hires. In addition, the results tend to be more pronounced when firm age categories are considered. The estimates reveal that a worker in a 0-1 year old firm has approximately 49% less net worth than a worker in an 11+ year old firm, holding all else fixed.[35] The analogous difference is about 33% for a 4-5 year old firm. For recent hires, the corresponding differences are 23% and 13%, respectively.

Overall, the estimates in Table 11 support the model's prediction of worker sorting based on assets. Not only the workers in younger or smaller firms tend to have lower net worth, but also the recent hires in these firms tend to have fewer asset holdings compared to their counterparts in older and larger firms.[36]

In addition, the results were also robust when different measures of assets were used. These additional results are available upon request.

[35] Note that (18) implies that the percent change in $\widetilde{a}_{it}$ going from $d_{kit} = 0$ to $d_{kit} = 1$ can be estimated as $100\,(\exp(\widehat{\alpha}_k) - 1)$. For values of a_{it} that are not too small one can write $\widetilde{a}_{it} \simeq \ln(2a_{it}) = \ln(2) + \ln(a_{it})$.Therefore, the percent change in $(a_{it} + \sqrt{a_{it}^2 + 1})$ approximates the percent change in a_{it}.

[36] Additional analysis indicates that the findings in Table 11 is robust to different definitions of net worth. For instance, including home value in net worth leads to similar conclusions. In addition, quantile regressions for the 25th and 50th percentiles also broadly yield qualitatively similar patterns, indicating that the differences in net worth tend to prevail in different parts of the net worth distribution, not just at the mean and median.

8 Conclusion

Entrepreneurial firms, which are typically young and small, disproportionately hire younger individuals who come from nonemployment and provide lower earnings to these individuals compared to more established firms. To understand what kind of workers match with entrepreneurial firms versus other firms, this paper proposed a dynamic model of entrepreneurship, which features labor markets for two sectors, entrepreneurial and corporate, that vary in labor market frictions. The two sectors also possess different production technologies and face different financial constraints. These differences lead to a divergence in sectoral wages per unit of worker efficiency and induces a sorting of workers across the two sectors based on both productivity and wealth.

The calibrated model's equilibrium offers an answer to the question of who works for whom. Among individuals who look for work, less wealthy ones tend to take up job offers from the low-paying entrepreneurial sector, instead of waiting for a corporate job offer. This tendency results in a sorting of individuals across the two sectors by both wealth and productivity. The model is also able to account for the observed differences across the two sectors in employment shares, average worker earnings, and worker flows. The model's key prediction on worker sorting based on assets finds support in the data. The workers employed in young or small firms and those workers who were recently hired by these firms possess, on average, fewer assets than their counterparts in more established firms.

Analysis of the model reveals that labor market frictions play an important role in the model's ability to replicate key features of the entrepreneurial sector and to generate worker sorting based on assets and productivity. The model provides a rich framework to study the effects of various search frictions and financial constraints on the entrepreneurial sector and its workers. One avenue for future work is to assess the effect of changes in these frictions and constraints on the long-run decline of entrepreneurship in the U.S., and the implications of the decline on the earnings and wealth of workers who tend to take jobs in entrepreneurial firms.

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Appendix

A Algorithm for Solving The Model's Equilibrium

A stationary equilibrium of the model is computed using an algorithm based on Huggett and Ventura (1999). The algorithm finds an equilibrium by iterating over value functions and decision rules over a discretized state space. Discretization of the continuous worker and entrepreneurial ability processes in (1) and (2) is done using the Tauchen (1986) algorithm with a 21-point support for the distribution implied by the process. The support is bounded below and above the mean by 2.5 times the standard deviation. The asset grid is discretized to 201 points. The spacing between points on the asset grid increases with asset levels. Asset gridpoints are placed according to $a_1 = 0$, $a_j = \psi j^\chi$ for $j = 2, ..., 201$, where $\chi = 3.0$, $\psi = \bar{a}/(201^\chi)$ and $\bar{a}$ is an upper bound. The algorithm is as follows.

1. Guess a value for the capital-labor ratio in the corporate sector, K/L,
2. Calculate the values $w_f = (1-\nu)AK^\nu L^{-\nu}$ and $r = \nu AK^{\nu-1}L^{1-\nu} - \delta$,
3. Set the initial value for the entrepreneurial sector wage equal to the corporate sector wage: $w_e = w_f$,
4. Calculate the optimal decision rules $c^i(s)$, $a^{i\prime}(s)$, $h^d(\tilde{s})$, $m^d(\tilde{s})$, $k(s), l(s)$, $(i, d \in \{n, f, e, m\})$
5. Calculate K'/L', $\int l(s)d\Psi^m(s)$, and $\int zd\Psi^e(s)$ implied by the optimal decision rules,
6. If the values of $|K'/L' - K/L| < \delta$ and $|\int l(s)d\Psi^m(s) - \int zd\Psi^e(s)| < \eta$ for some small $\delta > 0$ and $\eta > 0$ then a stationary equilibrium has been found. Otherwise, update K/L and w_e, and repeat steps 4-6.

The parameter vector $M = \{\alpha, \lambda, \gamma, \phi_e, \phi_f, \mu\}$ is recovered using a Nelder-Mead Simplex algorithm, where the objective function is set to minimize the distance between the target and simulation moments. Target moments m_j $(j = 1, ..., J)$ are described in Section 4. The objective function used is

$$\sum_{j=1}^{J}[(log(m_j/d_j))^2],$$

where m_j is the moment j calculated using simulated data in the stationary equilibrium and d_j is the corresponding data moment. The Fortran 90 code for the simplex algorithm is taken from the public domain and was written by Alan Miller.[37]

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B Estimation of the Parameters $\rho_\theta, \sigma_\theta$, and ξ

The estimation of the decreasing returns parameter, ξ, for entrepreneurial firms, and the parameters for the entrepreneurial productivity process, $\{\rho_\theta, \sigma_\theta\}$, is based on the framework of Abraham and White (2015).[38] The framework allows the estimation of the parameters $\{\rho_\theta, \sigma_\theta, \xi\}$ simultaneously. Consider a production function for a manufacturing firm i in the form of

$$y_{it} = \theta_{it} \left(k_{it}^{a_k} l_{it}^{a_l} x_{it}^{1-a_k-a_l} \right)^{\xi}, \tag{20}$$

which includes materials and energy, x_{it}, as an input, and a productivity process $\ln \theta_{it} = (1 - \rho_\theta)\mu_i + \delta_t + \rho_\theta \ln \theta_{it-1} + \epsilon_t$, where μ_i is a firm-specific productivity parameter, δ_t is a year effect that captures general changes in productivity that apply to all firms, and $\epsilon_t \sim N(0, \sigma_\theta)$. The parameters ρ_θ and σ_θ are allowed to vary across industries. The inclusion of the materials and energy in the production function controls for the use of intermediate inputs (materials and energy) in estimating the underlying total factor productivity process. The estimation also allows for a markup, η, common to all firms in an industry, which can be thought of as the average markup across firms that is assumed to be constant over time. Abraham and White (2015) estimate the parameters, ξ, ρ_θ and σ_θ in a GMM framework using the log-linear form of the production function and the Solow residual obtained from the gross output and cost shares of the inputs. See Abraham and White (2015) or Castiglionesi and Ornaghi (2013) for a derivation of the exact model estimated.

The data used for the estimation is the U.S. Census Bureau's Annual Survey of Manufactures (ASM), which provides an unbalanced panel of manufacturing establishments for the period 1972-2009. The data include, for each establishment, annual measures of output (value of shipments) and inputs (employment, materials/energy use, capital). This information is aggregated to the firm level. The age of the firm is also available, which is the age of the oldest establishment of the firm. The establishments included in the ASM sampling frame typically have size 20 employees or more, so the parameter estimates are not representative of very small firms. The model yields estimates of ξ, ρ_θ, and σ_θ for young versus old firms at the 4-digit SIC industry level. The estimated values for young firms are then averaged across industries to be used in the calibration of the baseline model. The analysis is limited to the manufacturing sector because of the unavailability of similar data for other sectors of the economy to calculate the revenue-based productivity of an establishment.

A remark is in order for how the estimated parameters of the three-input production function in (20) are used to calibrate the model's two-factor production function in (3). In the production function (20) used for estimation, the decreasing returns parameter, ξ, is the same for each of the

[38] Also see Castiglionesi and Ornaghi (2013) for a similar estimation methodology.

three inputs. Because the decreasing returns parameter is common to all inputs, in the model's calibration the estimated decreasing returns parameter $\xi = 0.88$ is applied to the two-factor production function in (3). Similarly, the total factor productivity process is not specific to any input (i.e. Hicks neutral) in (20). Therefore, the estimated productivity process based on the three-factor production function in (20) is assumed to apply to the two-factor production in (3).

Table 1. The fraction of entrepreneurs in the U.S. economy in 2000

Basis	**Fraction of entrep.**	**Non-entrep. Firm Pay Premium**	**Employment Share**	**Source**
Young and small firms (0-5 yr & emp $\leq$ 7)	1.7%	20.8%	3.6%	LBD
Young and small firms (0-5 yr & emp $\leq$ 15)	2.0%	18.5%	5.9%	LBD
Young firms (0-5 yr)	2.2%	17.2%	15.7%	LBD
Young (0-5 yr) + small old (6+ yr & emp $\leq$ 7) firms	4.9%	39.7%	20.8%	LBD
Young (0-5 yr) + small old (6+ yr & emp $\leq$ 15) firms	5.5%	44.7%	25.4%	LBD
Small firms (emp $\leq$ 10)	4.9%	33.5%	11.8%	LBD
Small firms (emp $\leq$ 20)	5.5%	36.7%	18.6%	LBD
Small firms (emp $\leq$ 25)	5.7%	37.4%	21.0%	LBD
Young firms (0-10 yr)	3.2%	16.6%	24.8%	LBD
Young (0-10 yr) + small old (11+ yr & emp $\leq$ 20) firms	5.8%	49.8%	33.1%	LBD
Firms classified with certainty as non-public	6.0%	45.2%	44.0%	SBO
Business owners with employees (Males 25-64)	2.9%	NA	NA	SIPP
Business owners with employees (Males 25-54)	2.8%	NA	NA	SIPP
Business owners with employees (All)	2.4%	NA	NA	SIPP
Business owners with employees (All 25-54)	2.3%	NA	NA	SIPP

Notes: The data sources are Longitudinal Business Database (LBD), Survey of Business Owners (SBO), and Survey of Income and Program Participation (SIPP). Estimates pertain to the year 2000. The denominator used to calculate fraction of entrepreneurs is the population of males aged 15-64 years, unless indicated otherwise. The calculations assume that each entrepreneurial firm is owned by a single entrepreneur.

Table 2. Some facts about entrepreneurial firms

Metric	**Young Firms** (0-5 years of age)			**Small Firms** ($\leq$ 20 employees)		
	1987	**2000**	**2012**	**1987**	**2000**	**2012**
Share of firms	0.48	0.41	0.33	0.90	0.89	0.90
Share of employment	0.21	0.16	0.11	0.21	0.19	0.18
Relative median of the firm-level average earnings	0.79	0.85	0.75	0.72	0.73	0.76
Relative average firm employment	0.28	0.26	0.25	0.03	0.03	0.02
Share of hires from nonemployment	–	0.24	0.26	–	0.26	0.27
Share of separations to nonemployment	–	0.21	0.23	–	0.26	0.27
Relative share of hires from nonemployment	–	1.05	1.04	–	1.20	1.18
Relative share of separations to nonemployment	–	1.03	1.05	–	1.24	1.20

Notes: The data sources are Longitudinal Business Database (LBD) and Longitudinal Employer-Household Database (LEHD). A young firm is defined as one that is 0-5 years old. A small firm is defined as one that has at most 20 employees. "Relative" indicates that the value is expressed relative to that of the firms that are not young (more than 5 years old) . or that are large (have more than 20 employees).

Table 3. The parameter values in the baseline model

Selected using a priori information or estimated using data

Parameter	**Value**	**Source**
Discount rate, β	0.98	Business cycle literature (Annual int. rate = 0.04)
Capital share in production, ν	0.36	Business cycle literature
Capital depreciation rate, δ	0.015	Business cycle literature (Annual rate = 0.06)
Productivity of the corporate sector, A	0.36	Normalization ($A = e^{-1}$)
Labor productivity, $\{\rho_z, \sigma_z\}$	$\{0.97, 0.13\}$	Heathcoate et al. (2010)
Entrepreneurial ability (Persistence), $\{\rho_\theta, \sigma_\theta\}$	$\{0.30, 0.18\}$	Estimated based on Abraham and White (2015)
Returns-to-scale in entrepreneurship, ξ	0.88	Estimated based on Abraham and White (2015)
Entrepreneur transition rate into corporate sector, p	0.001	IPO and Merger and Acquisition rates

Recovered using the calibration procedure

Parameter	**Value**	**Target**
Disutility from labor, α	0.82	Fraction employed–15-64 yrs old males (80%)
Job offer rate, λ	0.52	Job finding rate from nonemployment (45%)
Corporate sector job offer rate, γ	0.92	Share of employment in the corp. sector (0.84)
Job separation rates, $\{\phi_e, \phi_f\}$	$\{0.000, 0.012\}$	Separation rate from employment (1.9%)
Entrepreneurial ability (Mean), μ	0.24	Fraction of entrepreneurs (4.0%)

Notes: See Appendix B for the estimation of returns-to-scale for entrepreneurs and the parameters for the entrepreneurial ability process. Job separation and finding rates are taken from Shimer (2012). The fraction of entrepreneurs is based on the estimates in Table 1.

Table 4. The properties of the baseline model

Variable	**Model**	**Data**
Employment-to-population ratio	81%	80%
Share of employment (Entrepreneurial sector)	15%	16%
Fraction of entrepreneurs	5%	4%
Average worker productivity ratio (Corporate/Entrepreneurial)	1.17	NA
Corporate average earnings premium	19%	17%
Ratio of average worker assets (Corporate/Entrepreneurial)	2.3	1.2
Ratio of average worker assets in first quarter of job (Corporate/Entrepreneurial)	1.3	1.1
Employment-to-nonemployment (E-to-N) flow	1.9%	1.9%
Nonemployment-to-employment (N-to-E) flow	46%	45%
Interest rate, r	0.010	0.010
Wage ratio (w_f/w_e)	1.02	NA
Transfer rate, δ_τ	0.00003	NA

Notes: Employment-to-population ratio is based on the male population 15-64 years old. Share of employment in the entrepreneurial sector and corporate earnings premium are based on the Longitudinal Business Database (LBD). Fraction of entrepreneurs is based on the estimates in Table 2. The estimates for average worker assets are based on Survey of Income and Program Participation (SIPP)–see Section 7. E-to-N and N-to-E flows are taken from Shimer (2012).

Table 5. Inequality in the baseline model

	Share of Wealth		**Share of Earnings**	
Quintile	**Model**	**Data**	**Model**	**Data**
1st	0.9%	−0.5%	3.2%	7.5%
2nd	5.1%	0.5%	12.8%	11.3%
3rd	11.2%	5.0%	12.9%	18.7%
4th	21.4%	18.7%	16.6%	24.2%
5th	61.3%	76.2%	54.6%	38.2%

Notes: Wealth is measured by assets. The only negative value in the table is due to the high prevalence of negative assets at the bottom of the wealth distribution.

Table 6. The role of wages in worker sorting and equilibrium allocations

Variable	**Baseline**	**Equal Wages** ($w_e = w_f$)	**Equal Wages and Separation Rates** ($w_e = w_f, \phi_e = \phi_f$)
Employment-to-population ratio	82%	85%	83%
Share of employment (Entrepreneurial sector)	15%	33%	8%
Fraction of entrepreneurs	5%	3%	4%
Ratio of worker productivity (Corporate/Entrepreneurial)	1.17	1.15	1.00
Ratio of average worker assets (Corporate/Entrepreneurial)	2.30	1.90	1.00

Notes: The column "Equal Wages" pertains to the case where the entrepreneurial sector wage is set to be equal to the corporate sector wage in the baseline. The column "Equal Wages and Separation rates" pertains to the case where the entrepreneurial sector wage and separation rate are set to be equal to their corporate sector counterparts in the baseline model. All other parameters are fixed at their baseline values.

Table 7. Parameters under wage setting by the corporate sector

Parameter	**Baseline**	**Wage Setting by Corporate Sector**
Disutility from labor, α	0.82	0.84
Job offer rate, λ	0.52	0.51
Corporate sector job offer rate, γ	0.92	0.96
Entrepreneurial sector job separation rate, ϕ_e	0.000	0.000
Corporate sector job separation rate, ϕ_f	0.012	0.011
Entrepreneurial ability (Mean), μ	0.242	0.244

Notes: The model where wage is set by the corporate sector re-estimates all model parameters to match the data targets.

Table 8. The properties of the model with no entrepreneurial uncertainty

Variable	**Baseline**	**No Uncertainty**
Share of employment (Entrepreneurial sector)	15%	18%
Fraction of entrepreneurs	5%	5%
Share of entrepreneurs with binding borrowing constraint	0.90	1.00
Ratio of average worker productivity (Corporate/Entrepreneurial)	1.17	1.19
Corporate average earnings premium	19%	17%
Ratio of average worker assets (Corporate/Entrepreneurial)	2.3	2.1
Wage ratio (w_f/w_e)	1.02	0.98
Disutility from labor, α	0.82	0.82
Job offer rate, λ	0.52	0.52
Corporate sector job offer rate, γ	0.92	0.97
Entrepreneurial sector job separation rate, ϕ_e	0.000	0.000
Corporate sector job separation rate, ϕ_f	0.012	0.012
Entrepreneurial ability (Mean), μ	0.242	0.242

Notes: The column "No Uncertainty" refers to the model where the uncertainty about the initial draw of entrepreneurial ability is removed at the time entrepreneurship decision is made. All model parameters are re-estimated to match the data targets.

Table 9. The properties of the model with relaxed borrowing constraint

Variable	**Baseline** ($b = 1.5$)	**Higher Borrowing Limit** ($b = 2.0$)
Share of employment (Entrepreneurial sector)	15%	18%
Fraction of entrepreneurs	5%	5%
Share of entrepreneurs with binding borrowing constraint	0.90	0.92
Ratio of average worker productivity (Corporate/Entrepreneurial)	1.17	1.16
Corporate average earnings premium	19%	15%
Ratio of average worker assets (Corporate/Entrepreneurial)	2.3	1.9
Wage ratio (w_f/w_e)	1.02	0.99
Disutility from labor, α	0.82	0.82
Job offer rate, λ	0.52	0.53
Corporate sector job offer rate, γ	0.92	0.97
Entrepreneurial sector job separation rate, ϕ_e	0.000	0.000
Corporate sector job separation rate, ϕ_f	0.012	0.011
Entrepreneurial ability (Mean), μ	0.242	0.204

Notes: In the model with higher borrowing limit, all model parameters are re-estimated to match the data targets.

Table 10. Household net worth by firm age and size

	Mean		Quasi-median	
Firm Age (years):	**0-5**	**6+**	**0-5**	**6+**
Net worth (All workers)	$108,308	$125,506	$13,105	$24,121
s.e.	(2,491)	(853)	(455)	(260)
N	12,000	101,000	12,000	101,000
Net worth (Recent hire)	$90,921	$97,837	$9,396	$11,624
s.e.	(1,175)	(1,328)	(502)	(287)
N	6,000	27,000	6,000	27,000
Earnings (All workers)	$13,400	$16,601	$9,743	$12,284
s.e.	(181)	(600)	(93)	(35)
N	11,000	98,000	11,000	98,000
Earnings (Recent hire)	$10,505	$13,904	$7,452	$8,242
	(191)	(1,172)	(111)	(57)
N	5,000	23,000	5,000	23,000
Firm Size (employees):	**0-49**	**50+**	**0-49**	**50+**
Net worth (All)	$111,167	$127,801	$15,045	$25,777
s.e.	(1,620)	(930)	(392)	(305)
N	28,000	85,000	28,000	85,000
Net worth (Recent hire)	$91,214	$97,812	$10,181	$11,734
s.e.	(1,842)	(1,670)	(453)	(316)
N	10,000	22,000	10,000	22,000
Earnings (All)	$12,813	$17,383	$12,915	$15,028
s.e.	(144)	(711)	(48)	(38)
N	26,000	82,000	26,000	82,000
Earnings (Recent hire)	$10,022	$14,826	$7,300	$8,542
s.e.	(150)	(1,392)	(76)	(64)
N	9,000	19,000	9,000	19,000

Notes: Standard errors in parentheses. The number of observations N is rounded to the nearest 1000 for disclosure purposes. The sample is males aged 15-64 years. For quasi-median the standard error is calculated using bootstrap.

Table 11. Regression analysis of net worth

	All Workers		Recent Hires	
Firm Age Category	**OLS**	**Median Regression**	**OLS**	**Median Regression**
0-1 years	-0.675^{***}	-0.319^{***}	-0.262^{*}	-0.142^{**}
	(0.101)	(0.047)	(0.144)	(0.063)
2-3 years	-0.510^{***}	-0.236^{***}	-0.133^{*}	-0.036^{*}
	(0.096)	(0.040)	(0.066)	(0.020)
4-5 years	-0.403^{***}	-0.218^{***}	-0.140^{*}	-0.132^{*}
	(0.097)	(0.041)	(0.080)	(0.072)
6-10 years	-0.556^{***}	-0.215^{***}	-0.459^{***}	-0.197^{***}
	(0.061)	(0.022)	(0.118)	(0.055)
N	235,000	235,000	66,000	66,000
Firm Size Category				
0-19 employees	-0.310^{***}	-0.196^{***}	-0.063	-0.048
	(0.055)	(0.023)	(0.100)	(0.044)
20-49 employees	-0.461^{***}	-0.186^{***}	-0.152	-0.093^{*}
	(0.065)	(0.024)	(0.125)	(0.053)
50-249 employees	-0.365^{***}	-0.212^{***}	-0.333^{***}	-0.175^{***}
	(0.051)	(0.019)	(0.103)	(0.043)
250-499 employees	-0.440^{***}	-0.213^{***}	-0.188^{**}	-0.140^{**}
	(0.073)	(0.027)	(0.087)	(0.060)
N	236,000	236,000	67,000	67,000

Notes: Standard errors in parentheses. All regressions include the individual's gender, race, marital status, education level, age, and age-squared, as well as industry and year fixed effects. The number of observations N is rounded to the nearest 1000 for disclosure purposes. The regressions include all workers. The omitted category is 11+ years for the regressions with firm age, and 500+ employees for the regression with firm size. *,**, and *** indicate statistical significance at 10, 5, and 1% levels, respectively.

Figure 1. The distributions of entrepreneurial ability, assets, capital input and labor input – baseline model

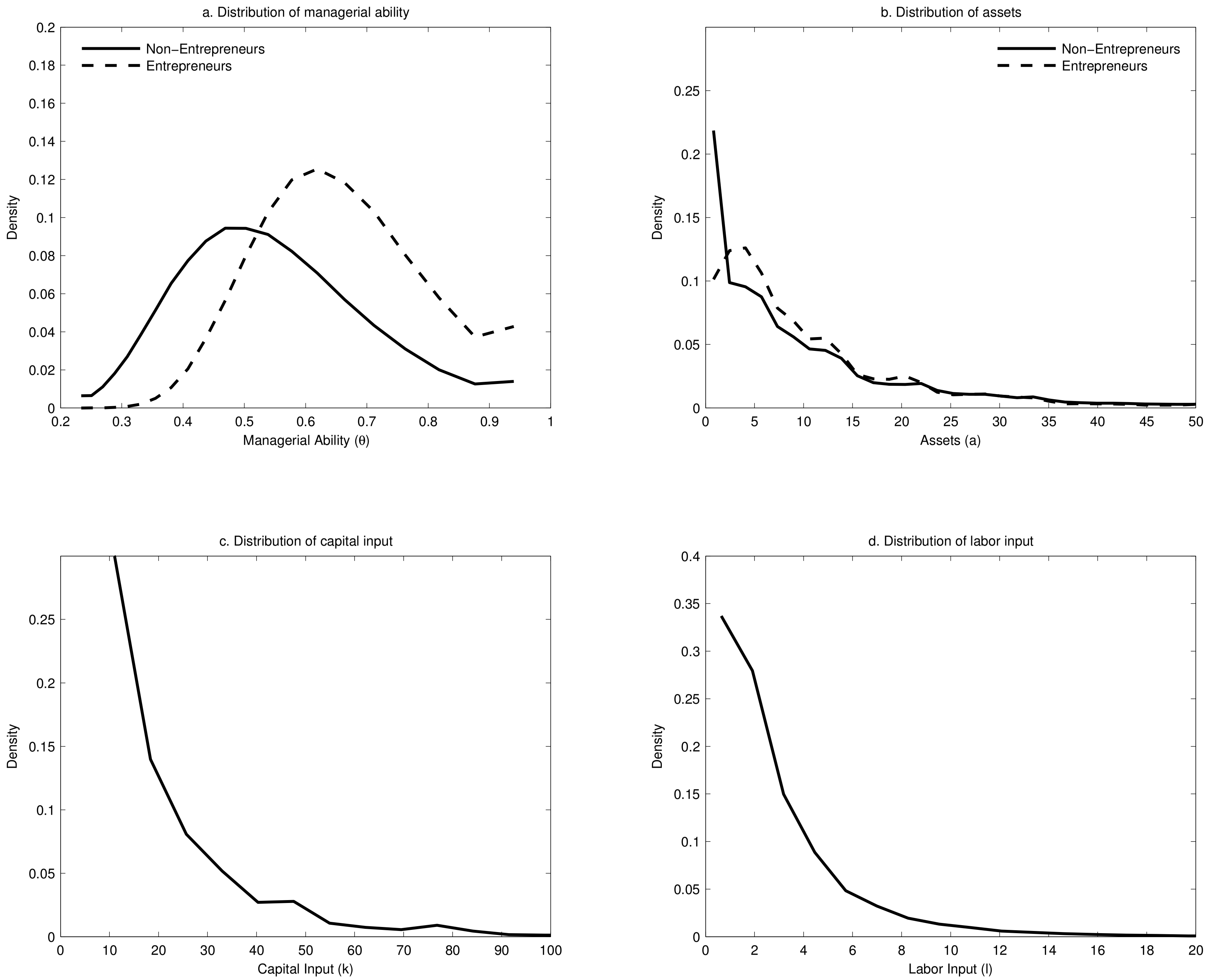

Figure 2. The allocation of individuals – baseline model

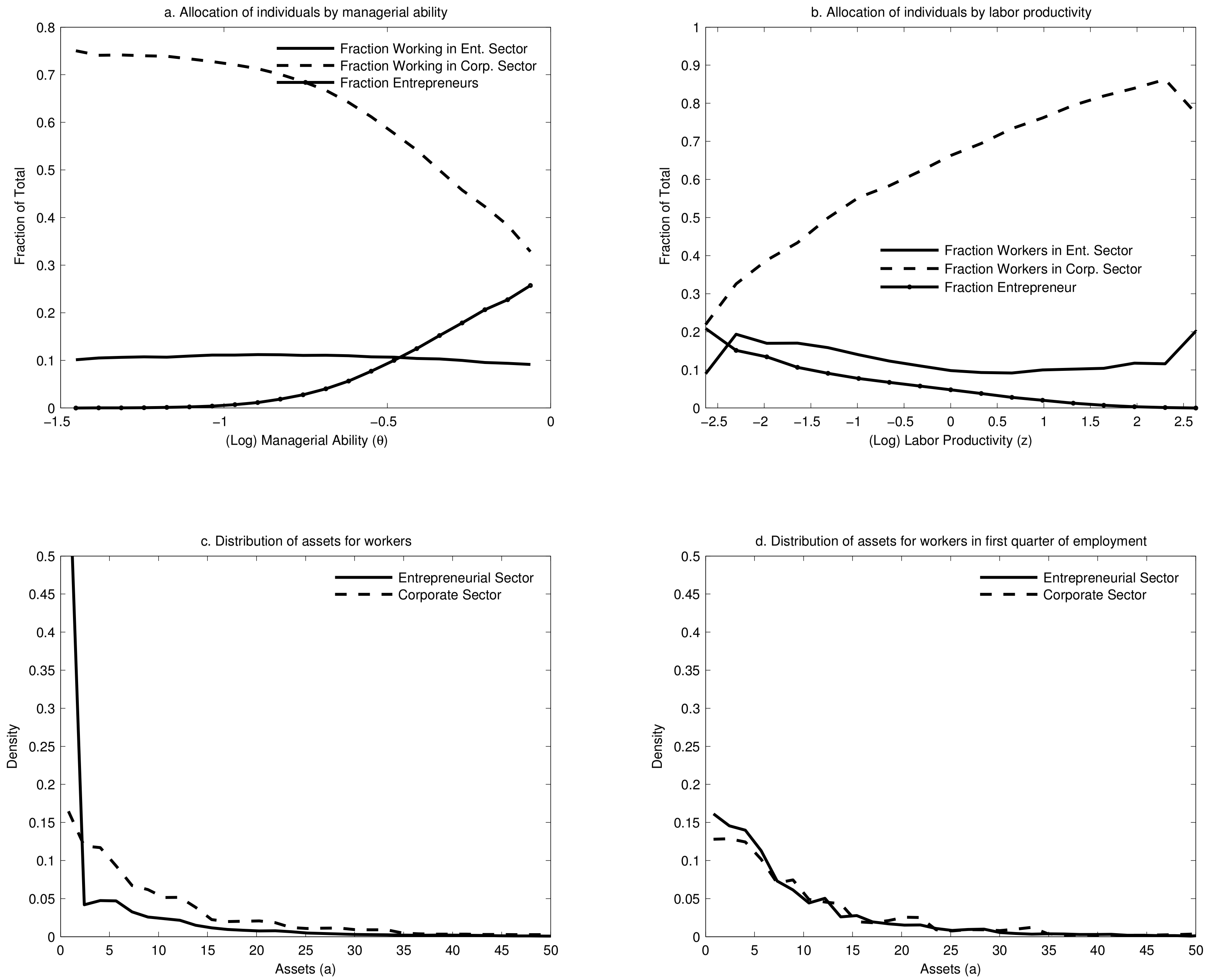

Figure 3. The value functions and decision rules – baseline model

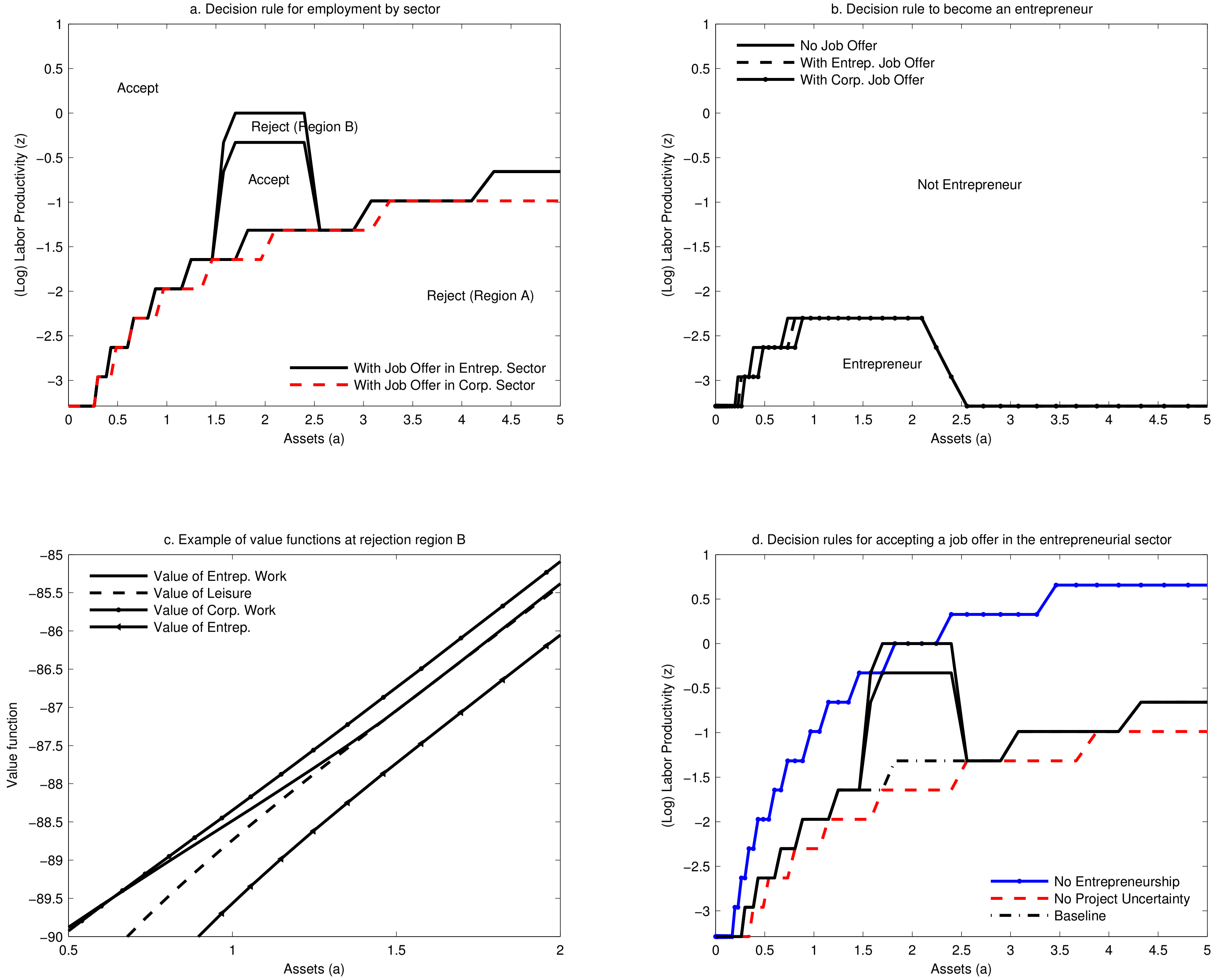

Figure 4: The Distribution of managerial ability among entrepreneurs

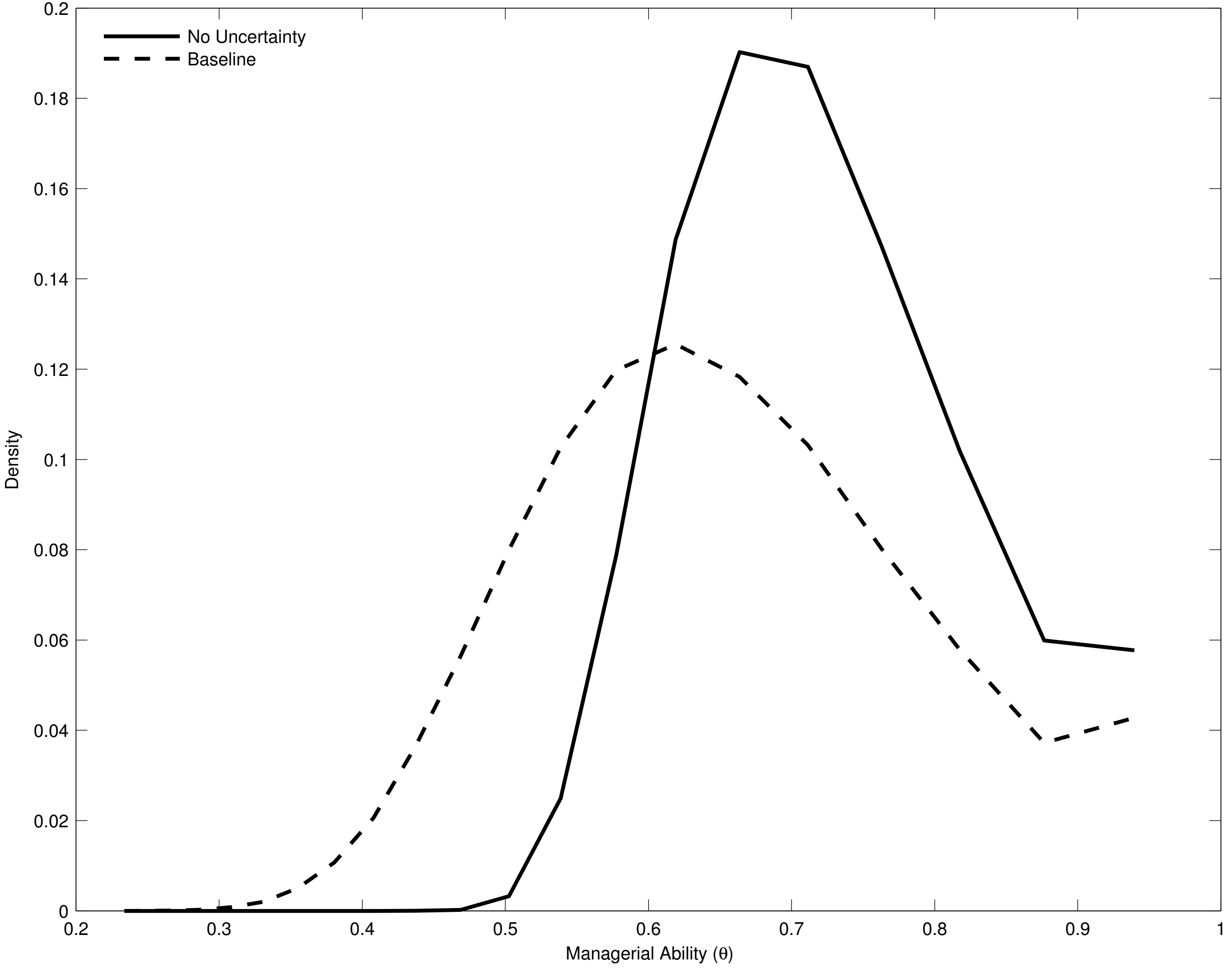

Figure 5. Experiments with corporate sector job offer rate – vertical dashed line indicates baseline value (0.92)