Who Works for Whom?
Worker Sorting in a Model of Entrepreneurship with Heterogeneous Labor Markets*

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Abstract

Compared to more established firms, young firms tend to hire younger workers and provide them with lower earnings. To understand these facts, a dynamic model of entrepreneurship is constructed, where individuals can become entrepreneurs, work in either a corporate or an entrepreneurial sector. Sectoral differences in production technology, financial constraints, and labor market frictions lead to sector-specific wages and worker sorting into the entrepreneurial sector by productivity and assets. Individuals with lower assets tend to accept jobs in the entrepreneurial sector, an implication that finds support in the data. The analysis indicates that sector-specific labor market frictions are critical to the model's ability to generate worker sorting and to match the key features of the entrepreneurial sector.

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*Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential data are disclosed.

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1 Introduction

Job creation by entrepreneurs is an important component of employment dynamics in the United States. In a typical year, new firm startups account for about 3% of total employment but almost 20% of gross job creation.[1] The jobs entrepreneurs create, however, may not always be the most desirable ones. Entrepreneurial firms, which are generally thought of as startups or young and small firms, provide lower earnings on average to their workers compared with older or larger firms.[2] They also tend to hire disproportionately from the pool of workers who are young and have lower education.[3] Jobs in entrepreneurial firms may therefore play an important role in the labor market by providing employment opportunities for those who would otherwise be nonemployed or wait for a higher-paying job offer from an established firm. Despite the increasing attention to differences in worker characteristics and earnings across entrepreneurial versus more established firms, the mechanisms by which workers sort into these two types of firms, and how this sorting is influenced by various labor market and financial frictions, remain relatively less understood.[4]

The long-run decline in business startups and diminished business dynamism also call for a better understanding of the connection between the supply of entrepreneurial firms and the market for the type of labor these firms attract.[5] The share of young employers in the population of firms has been falling, and workers are increasingly employed in older firms.[6] Businesses that have formed recently tend to create fewer jobs and pay lower wages, and the decline of business startups explains part of the decline in worker reallocation rates.[7] As a result of the decline, those individuals who tend to work for entrepreneurs may face an increasingly lower supply of new entrepreneurial jobs. Conversely, changing dynamics of labor markets may have consequences for entrepreneurs' ability to hire and retain workers, and hence, their cost of doing business. This

[1] At the same time, about 40% of the jobs created by startups disappear due to failure within 5 years of entry. See Haltiwanger et al. (2013).

[2] See, e.g., Brown and Medoff (1989) for the connection between firm size and earnings. Brown and Medoff (2003), Kölling et al. (2002), and Dinlersoz et al. (2013) document, among others, the connection between the age of an establishment or a firm, on the one hand, and the average earnings of workers, on the other.

[3] See, e.g., Ouimet and Zarutskie (2014) and Goetz et al. (2015).

[4] See Babina et al. (2017) for an alternative methodology to study worker sorting along these dimensions. See also Narita (2013) for a search-theoretic analysis of entrepreneurship in developing countries.

[5] For long-run trends in the number of new firm startups and young firms, see, e.g., Decker et al. (2014a). Recent work on the decline of entrepreneurship include Siemer (2014), Pugsley and Sahin (2015) and Karahan et al. (2015). These studies focus mainly on the decline in firm startups or age zero firms (a flow measure), which are a subset of the broader set of entrepreneurial firms (a stock measure) in the economy at any point in time, and abstract from labor and financial markets.

[6] See, e.g., Decker et al. (2014b) and Hathaway and Litan (2014).

[7] See, e.g., See Sedlacek and Sterk (2014) and Hyatt and Spletzer (2013).

connection between the supply of entrepreneurs and the supply of labor to their firms leads to several questions. What kind of individuals choose to work for entrepreneurs, and why? How do financial and labor market frictions affect the decision to become an entrepreneur and to work for one? Which type of frictions are critical in generating the observed allocation of workers to entrepreneurial businesses? These questions demand a framework where individuals face not only the decision to become entrepreneurs, but also the decision to work for entrepreneurial versus other firms.

This paper develops a model to study jointly the questions of who becomes an entrepreneur and what kind of workers sort into entrepreneurial firms in the presence of search frictions in the labor market and financial constraints for entrepreneurs. The calibrated model's equilibrium exhibits worker sorting: individuals with lower assets on average tend to take jobs in the entrepreneurial sector. However, workers with even moderate amount of assets accept entrepreneurial sector employment if their labor productivity is sufficiently large. This results in slightly higher average worker productivity in the entrepreneurial sector. The analysis also explores in detail the model's mechanisms that generate worker sorting and entrepreneurship in order to isolate the roles of financial constraints, labor market frictions, and entrepreneurial uncertainty. Labor market frictions – and sector-specific job finding rates, in particular – are central to generating differences in levels of worker assets between sectors. The analysis of a novel matched data on workers' assets and their employers provides some support for the implications of the model on worker sorting.

In the model, individuals differ in wealth, entrepreneurial ability, and worker productivity. Each individual can become an entrepreneur, or work in one of the two sectors: entrepreneurial and corporate–a label for the set of firms that don't face the constraints entrepreneurial firms do. The constraints entrepreneurs face are of two types. The entrepreneurial production is subject to diminishing returns that arise from the limits to entrepreneurs' span-of-control. In contrast, firms in the corporate sector can scale up production without such restrictions. In addition, entrepreneurs can borrow only up to a limit to operate their businesses–a constraint that does not apply to corporate sector firms.

The choice to become an entrepreneur and to work for one are endogenously determined, along with the price of labor entrepreneurs face. The match between workers and firms is subject to frictions in the labor market. Not all nonemployed individuals who look for a job can find one, and workers can be separated from their employers involuntarily, in addition to voluntary separations. These labor market frictions, however, are allowed to vary across the two sectors. Job offers arrive at different rates, and involuntary separations occur with different probabilities. Workers can also switch sectors without a spell of nonemployment through intersectoral "job-to-job" transitions. The differences across the two sectors in various frictions lead

to divergence in sectoral wages per unit of worker efficiency. This wage differential, combined with the heterogeneity in worker productivity and wealth, results in worker sorting across the two sectors based on both productivity and assets.

The model outlined above is related to recent models of entrepreneurship.[8] What distinguishes it from these models, however, is the presence of sector-specific labor market frictions and wages. The labor market frictions prevent costless movement of workers across sectors, and in and out of nonemployment. Because of these frictions, an equilibrium with two distinct wage rates emerges, where some workers take low-wage jobs in the entrepreneurial sector even though higher-wage corporate jobs are available. In particular, differences in labor market frictions are needed for the model's equilibrium to replicate observed facts. The labor market frictions, together with the differences in production technology and financial frictions across the two sectors, enable the model to generate employment shares, worker earnings, and worker flows to and from nonemployment that are consistent with their observed counterparts. At the same time, the model's equilibrium accounts for the observed fraction of entrepreneurs in the population, as well as the distributions of wealth for entrepreneurs and workers.

The model provides an answer to the central question of who works for whom. A key property of the model's equilibrium is that workers in the entrepreneurial sector tend to have fewer assets, lower earnings, and higher labor productivity compared to those in the corporate sector.[9] The asset differential is not only driven by the fact that individuals who work in the higher-wage corporate sector can accumulate on average more wealth over time than their counterparts in the entrepreneurial sector. There is an important selection effect: individuals who take jobs in the entrepreneurial sector tend to be less wealthy even *at the time* they take these jobs. In other words, the wealth and productivity differences across the two sectors also apply to individuals who have just taken jobs in these sectors. Nonemployed individuals with a job offer from the entrepreneurial sector have to decide whether to reject this offer and wait for an offer from the higher-wage corporate sector. Individuals with lower levels of savings prefer to take jobs in the entrepreneurial sector rather than waiting. This sorting emerges in the absence of any inherent preference for working in entrepreneurial firms, or any form of compensation other than the equilibrium wages these firms provide.

Worker sorting by assets turns out to be a robust feature of the model. The presence of sector-specific labor market frictions that favor job draws in the corporate sector is critical

[8] See, among others, Quadrini (2000), Cagetti and De Nardi (2006), Kitao (2008), Buera and Shin (2011), Buera et al. (2015), and Bassetto et al. (2015).

[9] The role of asset heterogeneity in generating differential labor market outcomes has also been explored recently by Eeckhout and Sepahsalari (2014), and Herkenhoff et al. (2015). A similar mechanism is also at work in theoretical study of Browning et al. (2007). While these studies include directed search in labor markets, they do not model an entrepreneurship versus work decision.

in obtaining worker sorting. In addition, the presence of sector-specific separation rates are important for the model to match all targeted moments. To understand the mechanisms behind sorting and equilibrium allocations, additional analysis is carried out to isolate the roles of the model's key elements. In particular, the analysis explores the specific roles of the distinct sectoral wages for labor, the differential labor market frictions, the extent of the borrowing constraint for entrepreneurs, and the uncertainty about the entrepreneurial ability at the time of the entrepreneurship decision. Worker sorting prevails to varying degrees under alternative assumptions about the nature of these key elements.

The model's prediction that workers with lower assets sort into entrepreneurial firms is taken to data. The test of this prediction requires data not only on individuals' assets, but also on their employment choices and the characteristics of their employers. While data on worker assets is available from a variety of sources, measuring workers' assets by employer type (e.g. employer size or age), and especially at the time when they start a job, is more challenging. The analysis uses a novel combination of data on workers' net worth from the Survey of Income and Program Participation (SIPP) and data at the worker-job level from the Longitudinal Employer-Household Dynamics (LEHD) program that captures employer characteristics and workers' job transitions. The empirical counterpart of the model's entrepreneurial firms are taken to be young firms.[10] The findings suggest that individuals who work in younger firms tend to have fewer assets than their counterparts in older firms. Furthermore, individuals who take jobs in young firms also tend to be less wealthy around the time they take these jobs, compared to those who take jobs in older, more established firms.[11] These findings support the predictions of the model on worker sorting based on assets.

The rest of the paper is organized as follows. The next section documents some key motivating facts about entrepreneurial firms. Section 3 introduces the model, followed by its baseline calibration in Section 4. The properties of the baseline model are discussed in Section 5. Section 6 explores the role of the model's key ingredients in determining equilibrium allocations and worker sorting. Sensitivity analysis with respect to some of the key assumptions of the model is carried out in Section 7. Section 8 offers empirical evidence on the predictions of the model on worker sorting by assets. Section 9 concludes.

[10]The findings are similar to a large extent when small firms are used instead, as discussed in the empirical analysis.

[11]Herkenhoff et al. (2015) provide empirical evidence that, among workers who undergo an involuntary job separation, those with more credit card debt spend less time in unemployment and accept lower wages, which is consistent with the evidence here that young and small (i.e., lower-paying) firms disproportionately hire workers with lower assets.

2 Motivating Observations

This section documents briefly some facts about entrepreneurial firms to motivate the model and its analysis. Entrepreneurial firms are often thought of as privately-held, young, and small firms.[12] Although not all young and small firms are entrepreneurial in nature (e.g. new businesses created by established firms), and some entrepreneurial firms may be young and large, firm age and size are frequently used to approximate the population of entrepreneurial businesses.[13] However, alternative definitions can be provided.

Table 1 presents various definitions of entrepreneurial firms and some key statistics associated with these firms for the year 2000. In all definitions, non-employer businesses are excluded, as the focus of this paper is on entrepreneurs who create jobs. In addition, each firm is assumed to have a single owner.[14] Assuming that the pool of potential entrepreneurs is the population of males aged 15-64 years in 2000, the fraction of entrepreneurs in the economy can then be approximated by the ratio of the number of entrepreneurial firms to the population of the pool.[15]

One way to define entrepreneurial firms is to apply various age and size criteria to the universe of employer-businesses in the U.S. Census Bureau's Longitudinal Business Database (LBD). Based on these criteria, Table 1 reveals that the fraction of entrepreneurs ranges from a rather conservative estimate of 1.7% to a less stringent one of 5.8%. Alternatively, one can define entrepreneurial firms as those that are not publicly owned and that have indicated some ownership demographics in the U.S. Census Bureau's Survey of Business Owners (SBO). This approach yields an estimate of 6.0%. To provide another set of estimates, one can use the responses to the question regarding employer-business ownership in the Survey of Income and Program Participation (SIPP). The estimates in this case vary from 2.3% to 2.9%. Table 1 also indicates that employment share of entrepreneurial firms varies between 3.6% to 44.0% across alternative definitions. Various definitions of entrepreneurial firms also imply a non-entrepreneurial firm average earnings premium in the range 16.6% to 49.8% – average worker earnings premium is defined as excess average worker earnings in non-entrepreneurial firms expressed as a percentage of the average worker earnings in entrepreneurial firms.

[12] The authors' calculations based on the 2007 Survey of Business Owners confirms this view of entrepreneurial firms. Of young businesses (less than 5 years old), 69.1% are owned by households, while such household-owned businesses account only 45.7% of the employment in these young firms. Of small firms (less than 50 employees), 88.4% are owned by households, and such businesses account for 40.0% of the employment in small firms.

[13] See, e.g., Haltiwanger et al. (2013).

[14] The datasets used to construct Table 1 have different definitions of business ownership. For example, household surveys count business owners who may operate multiple businesses, while business-level surveys often do not identify which businesses have owners who also own other businesses.

[15] In 2000, the population of males aged 15-64 years amounted to approximately 93 million based on U.S. Census Bureau's American Fact Finder.

The model in the next section studies how the differences in labor market frictions, financial constraints and technology give rise to worker productivity, earnings, and wealth differentials across the two types of firms. While alternative definitions of entrepreneurial firms are explored in Table 1 for establishing the robustness of the motivating facts, the rest of the analysis adopts an empirical definition of an entrepreneurial firm as a young firm of age 0-5 years. Adhering to this definition ensures consistency, to the extent possible, in the presentation of the subsequent findings.

3 The Model

Based on the differences between entrepreneurial and other firms highlighted in the previous section, the model considers an economy with two sectors: entrepreneurial and corporate. The sectors differ in production technologies, labor market frictions, and financial constraints. The model extends the framework of incomplete markets with occupational choice in the spirit of Quadrini (2000) and Cagetti and De Nardi (2006) to include heterogeneous labor markets, as in the "islands" economy of Lucas and Prescott (1974).[16] It also features indivisible labor choice characterized by frictions between production and leisure "islands", as in Krusell et al. (2011).

3.1 The Setup

The model is based on an overlapping generations framework that focuses on the decisions of prime age workers who start their working lives with no assets. The model abstracts from asset accumulation for retirement and any substantial role of government transfer programs to highlight the role of worker sorting among workers for whom these additional features are of secondary concern.

Time is denoted by t, and the discount factor is $\beta \in (0, 1)$. Individuals survive from one period to another with probability p_s. Individuals that die are replaced with newborns, so there is always a unit mass of individuals in the economy. Each period an individual is endowed with one unit of time, which can be used for production as a worker or an entrepreneur. Individuals have identical preferences represented by the period utility

$$u(c_t, h_t) = \ln c_t - \alpha h_t,$$

where $c_t \geq 0$ is the consumption, $\alpha > 0$ is the disutility from labor, and $h_t \in \{0, 1\}$ is an indicator of participation in the labor market as a worker or entrepreneur.[17]

[16] See also Alvarez and Veracierto (2000).

[17] The index for an individual is suppressed for notational simplicity.

Each individual possesses an amount, $a_t \geq 0$, of assets. Assets are not transferred across generations. Instead, assets from individuals that die are redistributed lump-sum equally across all surviving agents. Individuals also differ in their ability (or productivity), both as a worker and an entrepreneur. Worker productivity is summarized by $z_t > 0$ – the efficiency units of labor an individual can supply in a period. The productivity, z_t, evolves over time independently across individuals according to the process

$$\begin{aligned} \ln z_t &= \rho_z \ln z_{t-1} + \epsilon_t^z, \\ \epsilon_t^z &\sim N(0, \sigma_z^2). \end{aligned} \tag{1}$$

Similar to the worker ability, the entrepreneurial ability, θ_t, is also subject to random fluctuations independently across individuals

$$\begin{aligned} \ln \theta_t &= (1 - \rho_\theta)\mu + \rho_\theta \ln \theta_{t-1} + \epsilon_t^\theta, \\ \epsilon_t^\theta &\sim N(0, \sigma_\theta^2). \end{aligned} \tag{2}$$

For newly-born agents, both worker and entrepreneurial ability are drawn from the stationary distribution implied by the processes defined above.

Production takes place in corporate and entrepreneurial sectors, denoted by $j \in \{e, f\}$, respectively. The sectors possess different technologies. There is a representative firm in the corporate sector. It generates output, Y_t, by combining capital, K_t, and efficiency units of labor, L_t, through a constant-returns-to-scale production technology

$$Y_t = AK_t^\nu L_t^{1-\nu},$$

where $\nu \in (0, 1)$, and $A > 0$ is the corporate sector's total factor productivity.

Each firm in the entrepreneurial sector is operated by an entrepreneur with ability θ_t, who uses capital, k_t, and efficiency units of labor, l_t, to produce output, y_t, via a decreasing-returns-to-scale technology

$$y_t = \theta_t (k_t^\nu l_t^{1-\nu})^\xi, \tag{3}$$

where $\xi \in (0, 1)$ is a span-of-control parameter, which reflects the diminishing returns to the entrepreneur's managerial ability. Entrepreneurs also face a constant, exogenous probability of having their business bought out by the corporate sector. This feature captures, in a reduced form, the transition of firms from the entrepreneurial sector to the corporate sector.

There are two types of frictions. The first type pertains to the labor markets. Employment opportunities for nonemployed individuals arrive every period with a constant probability. Job offers can come from the corporate sector or the entrepreneurial sector. Let γ_j denote the employment offer probability from sector $j \in \{e, f\}$. Every period workers can separate from

their employers either voluntarily or involuntarily. The latter occurs with probability ϕ_j. Those individuals who are separated from firms or quit entrepreneurship transition into nonemployment and stay there for at least one period before receiving offers (if any), and face the decision to work or become an entrepreneur again.

Employed individuals can also transition between sectors without a spell of nonemployment. They receive a "job-to-job" transition offer with probability γ_j^q. Transitions are exogenous in the sense that workers cannot remain in their former sector of employment if they receive a transition offer.[18] The frictions in the labor market are thus completely specified by the set of parameters $\{\gamma_e, \gamma_f, \gamma_e^q, \gamma_f^q, \phi_e, \phi_f\}$.

The second type of frictions is financial in nature. There are borrowing constraints for entrepreneurs, and individuals are not allowed to carry negative assets, $a_t \geq 0$. Each period, an entrepreneur with assets, a_t, can access an amount of capital $k_t \leq ba_t$, where $b \geq 1$ is an exogenously given borrowing limit. When $b = 1$, entrepreneurs can only use their accumulated assets to finance production in any given period. The parameter b is the only parameter that governs the financial frictions for entrepreneurs. Capital rental rate is $r > 0$, and the depreciation rate is $\delta \in (0, 1)$.

The timing of events within a period is as follows. Individuals first realize their current-period labor productivity, z. Each nonemployed individual then receives a job offer from one of the sectors, and an employed individual receives a job offer for transitioning from one job to another. All individuals then make their decisions about whether to work, become an entrepreneur, or not work. Following this decision, entrepreneurs realize their current-period ability, θ, and choose their inputs for production. Each individual subsequently chooses how much to consume and save. At the end of the period, some individuals get hit by a mortality shock. They are replaced in the following period with individuals that have no assets. Some of the surviving employed individuals get separated from their employers exogenously. At the same time, some entrepreneurs' businesses are bought out by the corporate sector with exogenous probability, p. The entrepreneurs whose businesses transition into the corporate sector become nonemployed in the next period.

3.2 Individuals' Problems

Consider a stationary environment where policies and payoffs do not depend on calendar time. Let $s = (a, z, \theta)$ summarize an individual's assets, and worker and entrepreneurial ability in a period. In addition to s, each individual is differentiated by current-period location or

[18] This simplification forces bilateral flows between sectors and does not add to the computational complexity of the problem. In fact, the expectation of employment conditional on a job-to-job transition offer is the same as for a nonemployed worker.

island, which can be nonemployment (n), working in the corporate sector (f), working in the entrepreneurial sector (e), or being an entrepreneur (m).

The value of nonemployment is defined as

$$V^n(s) = \max_{c,a'\geq 0}(\ln c + \beta p_s \mathbb{E}_{z'|z}[\sum_{j\in\{e,f\}} \gamma_j \max\{\tilde{V}^j(s'), \tilde{V}^n(s'), \tilde{V}^m(s')\} \\ + (1-\gamma_e-\gamma_f)\max\{\tilde{V}^n(s'), \tilde{V}^m(s')\}]) \tag{4}$$

subject to the budget constraint

$$c + a' = (1+r)a + T,$$

where the lump-sum transfer to the individual, T, is financed by the assets of the individuals who die in the current period. Equation (4) reflects the fact that a nonemployed individual obtains the utility from consumption in the current period, and the expected value in the next period depends on whether a job offer is received, and the sector this offer comes from. The value functions $\tilde{V}^i(s')$, $i \in \{n, f, e, m\}$ give the expected value of being in location i in the next period, i.e. $\tilde{V}^i(s') = E_{\theta'|\theta}[V^i(a', z', \theta)]$.

Let w_j be the wage per unit of worker efficiency in sector $j \in \{e, f\}$. The value of an individual who works in sector j is given by

$$V^j(s) = \max_{c,a'\geq 0}(\ln c - \alpha + \beta p_s \mathbb{E}_{z'|z}[(1-\phi_j)[(1-\sum_{i\in\{e,f\}} \gamma_i^q)\max\{(\tilde{V}^j(s'), \tilde{V}^n(s'), \tilde{V}^m(s')\} \\ + \sum_{i\in\{e,f\}} \gamma_i^q \max\{(\tilde{V}^i(s'), \tilde{V}^n(s'), \tilde{V}^m(s')\}] + \phi_j \max\{\tilde{V}^n(s'), \tilde{V}^m(s')\}]) \tag{5}$$

subject to

$$c + a' = w_j z + (1+r)a + T,$$

Equation (5) indicates that an employed individual receives a current utility from consumption minus the disutility of work. In the next period, the individual's expected value depends on whether he gets separated. In the event of no separation, the individual can continue employment or quit to nonemployment or entrepreneurship. Continued employment is also subject to an exogenous job-to-job transition shock. In the event of separation, the individual transitions to nonemployment where he can choose to remain, or become an entrepreneur.

Finally, the value of an entrepreneur is

$$V^m(s) = \max_{c,a'\geq 0}(\ln c - \alpha + \beta p_s \mathbb{E}_{z'|z}[(1-p)\max\{\tilde{V}^n(s'), \tilde{V}^m(s')\} \\ + p\tilde{V}^n(a' + \tau(s), z', \theta')]) \tag{6}$$

subject to

$$c + a' = \pi(s) + (1+r)a + T,$$

where the entrepreneurial profit, $\pi(s)$, is given by

$$\pi(s) = \max_{l \geq 0, 0 \leq k \leq ba} \theta(k^{\nu} l^{1-\nu})^{\xi} - w_e l - (r + \delta)k. \tag{7}$$

The entrepreneurial value in (6) consists of the current period utility, and the next period's expected value, which depends on whether the entrepreneur's business is bought out by the corporate sector. The individual can continue to be an entrepreneur or choose to be nonemployed in the event the corporate sector does not purchase the entrepreneur's firm (with probability $1 - p$). If the firm is purchased (with probability p), the entrepreneur receives a transfer of $\tau(s)$ and becomes nonemployed in the following period.[19] The transfer, $\tau(s_t)$, is equal to the present discounted value of future stream of profits of the entrepreneur starting from the period of transfer, t

$$\tau(s_t) = \sum_{j=1}^{\infty} \left(\frac{1}{1+r} \right)^{j} \mathbb{E}_{\theta_{t+j}|\theta_{t+j-1}}[\pi^f(s_{t+j})]. \tag{8}$$

In other words, the corporate sector pays a transfer to an entrepreneur with state s that equals the expected value from perpetual operation of the entrepreneurial technology at its optimal scale by the corporate sector starting from period t. Note the discount rate $1/(1+r)$ on the right hand side of (8), as opposed to the subjective discount rate, β, of individuals. The former rate is the relevant one here since the corporate sector uses capital to purchase an entrepreneurial business, and the capital rental rate is r.

The optimal scale operated by the corporate sector is determined by the following profit maximization problem

$$\pi^f(s_{t+j}) = \max_{k,l \geq 0} \theta_{t+j}(k^{\nu} l^{1-\nu})^{\xi} - w_f l - (r + \delta + \delta_{\tau})k.$$

There is no upper bound on capital in this maximization problem, since the corporate sector faces no financial frictions, unlike the entrepreneurial sector. This simplified optimization problem ensures that the transfer can be calculated without considering future asset states or employment transitions of the entrepreneur. In equilibrium, the total amount of transfers from the corporate sector to the entrepreneurs is financed by a portion, δ_{τ}, of the capital used by the corporate sector, K. The effect of this transfer is to increase the marginal cost of capital in the corporate sector by δ_{τ}.

[19] The fact that p is exogenous and does not depend on entrepreneurial ability is a simple approach to formulating the transition of an entrepreneurial firm to the corporate sector. One could think of a more elaborate formulation where p is a function of the state s. However, the goal here is not to provide a model of transition over the life-cycle of an entrepreneur. In practice, the way this transition is modelled makes little difference in the framework here as long as such transitions are rare events – for instance, the low frequency of IPOs and M&A activity in the U.S. economy supports this view, as discussed in the model's calibration.

3.3 Equilibrium

Let $i \in \{n, f, e, m\}$ denote the labor status of an individual in any given period. In addition, let $d \in \{n, f, e, m\}$ be the "island" or "location" of the individual at the end of the previous period. Denote the remaining state space of agents making their labor choice decisions by $\tilde{s} = (a, z, \theta_{-1})$, where θ_{-1} denotes the previous period's entrepreneurial ability. A stationary competitive equilibrium for the model is a collection of value functions, $V^i(s)$, wage in each sector, w_j for $j \in \{e, f\}$, an interest rate, r, transfers, δ_τ and T, labor supply rules, $h^d(\widetilde{s})$, decision rules to become an entrepreneur, $m^d(\widetilde{s})$, saving and consumption rules, $a'^i(s)$ and $c^i(s)$, an entrepreneur's capital and labor choice rules, $k(s)$ and $l(s)$, and measures of individuals by labor status, $\Psi^i(s)$, such that

1. The saving and consumption rules, $a'^i(s)$ and $c^i(s)$, labor supply rules, $h^d(\widetilde{s})$, and the decision rules to become an entrepreneur, $m^d(\widetilde{s})$, solve the individuals' problems defined in (4), (5), and (6),

2. The interest rate, r, and the corporate sector wage, w_f, satisfy

$$r = \nu A K^{\nu-1} L^{1-\nu} - \delta - \delta_\tau, \tag{9}$$

$$w_f = (1-\nu) A K^{\nu} L^{-\nu}, \tag{10}$$

3. The transfer rate, δ_τ, ensures that total amount of transfers to entrepreneurs are accounted for by a portion of the corporate sector capital

$$\int p\tau(s) d\Psi^e(s) = \delta_\tau K, \tag{11}$$

4. The capital and labor choices, $k(s)$ and $l(s)$, solve the entrepreneur's problem in (7),

5. The measures, $\Psi^i(s)$, are consistent with the transitions of the individuals across islands,

6. Lump-sum transfers satisfy

$$T = (1 - p_s) \sum_i \int a(s) d\Psi^i(s),$$

7. Labor, capital, and goods markets clear

$$\int l(s) d\Psi^m(s) = \int z d\Psi^e(s), \qquad \text{(entrepreneurial sector labor)} \tag{12}$$

$$L = \int z d\Psi^f(s), \qquad \text{(corporate sector labor)} \tag{13}$$

$$K + \int k(s) d\Psi^m(s) = \sum_i \int a d\Psi^i(s), \qquad \text{(capital)} \quad (14)$$

$$Y + \int y(s) d\Psi^m(s) = \sum_i \int c(s) d\Psi^i(s) + \delta_\tau K + \delta \left(K + \int k(s) d\Psi^m(s) \right). \quad \text{(goods)} \quad (15)$$

where $y(s)$ denotes the output of an entrepreneur with state s.

While the corporate sector wage, w_f, depends on the representative corporate firm's labor choice (10), the entrepreneurial sector wage, w_e, is the value that equates the labor demand by all entrepreneurs to the labor supply of all workers in the entrepreneurial sector – equation (12). The amount of capital used by the two sectors must equal the total assets of all individuals in the economy, as ensured by (14). Finally, the total output of the economy must account for the total consumption by individuals, the replacement of the depreciated capital, and the transfers to entrepreneurs from the corporate sector, as shown in (15). The supplemental Online Appendix A outlines the algorithm that is used to solve for the stationary equilibrium numerically.

4 Calibration

The parameter values used in the calibration of the baseline model are shown in Table 2. Each period corresponds to one quarter. The discount rate, β, is set to 0.99, to match an annualized interest rate of 4%. Individuals' working life-time is chosen to approximate ages 15-64, which implies $p_s = 0.995$. The process for labor productivity in (1) is assigned the quarterly counterparts of annual parameters estimated by Heathcote et al. (2010). The annual parameters are $\{\rho_z, \sigma_z\} = \{0.97, 0.13\}$.[20]

The annual values of the parameters $\{\rho_\theta, \sigma_\theta\}$ of the process for managerial ability in (2) and the returns-to-scale parameter, ξ, are estimated separately for entrepreneurial firms (young firms aged 0-5 years) versus non-entrepreneurial firms (older firms aged 6+ years) in the manufacturing sector. The unavailability of data on inputs other than labor precludes the estimation of these parameters for firms in other sectors of the economy. The estimation follows the econometric methodology used in Abraham and White (2015), which allows joint estimation of the parameters $\{\rho_\theta, \sigma_\theta, \xi\}$ based on Castiglionasi and Ornaghi (2013) – see Online Appendix B.[21] The framework of Abraham and White (2015) has a number of desirable features. Notably, it allows for heterogeneity in the parameters $\{\rho_\theta, \sigma_\theta, \xi\}$ across industries. Abraham and White (2015) demonstrate that restricting these parameters to be the same across industries can lead

[20] The support of the labor productivity process is discretized using 21 grid points based on the Rouwenhorst method – see Kopecky and Suen (2010).

[21] See also the earlier version, Abraham and White (2006). The continouous process is approximated by 21 grid points underlying a discrete transition matrix using Tauchen (1986) method.

to upward bias in the estimate of the persistence parameter, ρ_θ.[22] The estimated parameters for the entrepreneurial ability process for θ at an annual rate turn out to be $\{\rho_\theta, \sigma_\theta\} = \{0.3, 0.18\}$, which are the averages across narrowly defined industries at the level of 4-digit SIC codes. The span-of-control parameter for young firms, ξ, has an average estimated value of 0.88 across industries. This value is smaller than the corresponding one for old firms (around 0.97), suggesting a lower span-of-control for entrepreneurial (young) firms.

Following Kitao (2008) and Buera and Shin (2011), the borrowing constraint parameter, b, is set to 1.5, implying that an entrepreneur can borrow up to 50% of his assets at the beginning of the period. Based on the business-cycle literature, the capital's share of output, ν, is set to 0.36, and the quarterly depreciation rate, δ, is taken to be 0.015, which corresponds to an annual depreciation rate of 0.06. The productivity of the corporate sector, A, is normalized to $\exp(-1)$.

In the model, entrepreneurs face a constant probability, p, of transitioning into the corporate sector. One can think of this transition as any event that largely removes the financial and managerial constraints the entrepreneur faces. For instance, a highly-successful entrepreneurial firm may grow large enough to transcend its financial and managerial constraints, or it can be acquired by a corporate sector firm. The firm may also engage in an IPO, which allows further access to capital markets. Consider, for example, the case of an IPO. Based on the Compustat database, the number of publicly-traded firms in any given year during the period 2002-2012 varies in the range of 5 to 6 thousand. This set of firms represent about 0.1% of the entire set of employer businesses in the U.S. during the same period, which range around 5-6 million. Similarly, the number of announced mergers and acquisitions in the U.S. ranged approximately from 8 to 14 thousand per year during the period 2000-2015, indicating that at most a fraction of about 0.1%-0.2% of firms engage in this type of announced merger and acquisition activity.[23] Given these estimates, a value of $p = 0.001$ was chosen to approximate the flow of entrepreneurial firms into the corporate sector.

On the choice of a value for the parameter p, it is important to emphasize that the notion of transition of a firm from the entrepreneurial sector to the corporate sector is introduced only to generate a simple link between the two sectors. This transition allows some entrepreneurial firms to operate without scale and borrowing constraints. Getting rid of the financing and scale constraints as a result of the transition might be one ultimate goal of entrepreneurial businesses. This possibility is captured in a parsimonious way with probability p. However, the exact magnitude of p does not matter substantially in the model's ability to match the data

[22] Other recent approaches to estimating the productivity persistence parameter have generally found higher estimates of persistence. Online Appendix C.2 carries out robustness analysis using a higher persistence parameter.

[23] These figures are provided by Institute of Mergers and Acquisitions, available at https://imaa-institute.org/m-and-a-us-united-states/.

targets, as long as it is not too large. In fact, the analysis is robust to different choices of p. In fact, a value of $p = 0$ on the low end, or $p = 0.004$ on the high end, produce very similar results. This robustness is demonstrated in Table C.1 in Online Appendix C.

The remaining parameters, denoted by the set $M = \{\alpha, \gamma_e, \gamma_f, \gamma_e^q, \gamma_f^q, \phi_e, \phi_f, \mu\}$, are chosen to match different targets that constitute a system of non-linear equations. While these equations are simultaneous in nature and involve all relevant parameters of the model, each equation plays an instrumental role in setting a specific parameter. The values of the targets are chosen to be the average value of their empirical counterparts in the early 2000s. For the disutility of labor, α, the key target is the employment-to-population ratio (0.80) among males aged 15-64 years. Two other targets, the share of employment in non-entrepreneurial firms (84%) and the average worker earnings premium for these firms (17%), are important in pinning down a value for the job offer rates, γ_j. The job separation rates, ϕ_j, are set so that the aggregate job separation rate (employment-to-nonemployment flows) is 8.5% as a fraction of total employment, and the aggregate job finding rate (nonemployment-to-employment flows) is 34%, based on LEHD data.[24]

To further simplify the analysis, the job-to-job transition offer probabilities are specified as $\gamma_j^q = \bar{\gamma}\gamma_j$, $j \in \{e, f\}$; that is, the frictions governing job-job-flows are proportional to the frictions for job offer arrivals. The parameter, $\bar{\gamma}$, is set such that the model replicates the share of employment that corresponds to workers moving between young and old firms observed in the LEHD (1.6%). Finally, the fraction of entrepreneurs, 4%, is targeted in assigning a value to the entrepreneurial ability parameter, μ. This fraction is within the range of various estimates provided in Table 1 – see Online Appendix A for details of the calibration procedure.

Calibration yields job offer rates $\gamma_e = 0.07$ and $\gamma_f = 0.46$, which imply significant frictions that impede worker flows into the entrepreneurial sector relative to the corporate sector. The exogenous separation probabilities are $\phi_e = 0.05$ and $\phi_f = 0.10$ — workers are half as likely to be exogenously separated from employment in the entrepreneurial sector than their counterparts in the corporate sector. These separation shocks do not include quits, which are endogenously determined. Note that employment in the entrepreneurial sector does not correspond to any particular firm-worker match, but rather employment at any one of the firms within that sector. In other words, in the entrepreneurial sector the firm that employs any given individual is indeterminate. As a result, one way to interpret the relationship $\phi_e < \phi_f$ is that an individual employed in the entrepreneurial sector is more likely to find employment at any firm in that sector until there is a voluntary separation, compared to a worker in the corporate sector. The separation rates should therefore be thought of as separation shocks to employment in a sector that may correspond to continuous employment spells at one or more firms in that sector.

[24]See Hahn et al. (2018) for a description of the dataset used to construct these statistics.

Similarly, within-sector job-to-job transitions are indeterminate in this framework, since workers that are employed in a sector for more than one period may be transitioning across firms in that sector without any friction. However, job-to-job transitions that results in a change of sector are allowed. These transitions have counterparts in the data and result in a calibrated value of $\bar{\gamma} = 0.10$.

5 Properties of the Baseline Model

This section provides an evaluation of the model's ability to match the targets described in the previous section, as well as other moments that are not targeted. The key features of the calibrated model's equilibrium are shown in Table 3. Overall, the model comes close to matching the targeted values. It produces an employment-to-population ratio of 82.5%. Around 4.0% of individuals choose to become entrepreneurs. Entrepreneurs have a stock of assets that is on average 74% larger than that of workers as seen in Table 3, which is consistent with available data.[25] Furthermore, as shown in Figure 1a, individuals with a higher level of entrepreneurial ability tend to become entrepreneurs – the distribution of managerial ability for entrepreneurs stochastically dominates that for non-entrepreneurs, in a first order stochastic sense. Entrepreneurs also tend to have higher levels of assets (Figure 1b). The capital input for entrepreneurial firms exhibits a skewed distribution (Figure 1c). Similarly, the distribution of the labor input (in efficiency units) for the entrepreneurial firms in Figure 1d is also highly-skewed.[26] The features of the model discussed so far also emerge in recent models of entrepreneurship, indicating that the model is able to capture the salient aspects of these models.[27]

The model's main distinguishing aspect, heterogeneous labor markets, provides further insight to the functioning of the labor markets and the nature of worker sorting. In fact, the model captures the underlying worker allocations and prices across the two labor markets that drive the observed worker sorting. The model's equilibrium is broadly consistent with the behavior of the key metrics for the U.S. labor market. In the baseline model, 14.0% of the employees work for entrepreneurial firms, close to the data counterpart of 16.0%. The model also delivers a corporate earnings premium of 16.6% consistent with its observed value of 17.0%. Employed workers switch jobs between entrepreneurial and corporate firms at a rate of 1.9% in the model,

[25] Average worker and entrepreneur assets are calculated using SIPP data discussed in more detail in Section 8. The statistic was calculated on the entire SIPP sample before it was merged with LEHD data.

[26] This shape is in line with the typical shape of the firm-level distributions of labor input in empirical studies. However, note that the labor input in the model (worker efficiency units) is different from the employment measure (the number of workers) typically used in empirical studies of firm size.

[27] See, e.g., Quadrini (2000), Cagetti and De Nardi (2006), Kitao (2008), Buera and Shin (2011), Buera et al. (2015), and Bassetto et al. (2015).

compared with 1.6% in the data.

The sectoral average worker earnings that generate the corporate earnings premium depend on the distribution of worker productivity (z) in each sector, as well as the wages per efficiency units of labor (w_e, w_f). The values for w_e and w_f are recovered as part of the calibrated model's equilibrium, but there is no observable target to discipline their values. The wage per efficiency unit of labor in the corporate sector turns out to be 23% higher than that in the entrepreneurial sector. Despite this wage premium, workers in the corporate sector are 5% less productive, on average, than a worker in the entrepreneurial sector. The sorting of individuals based on productivity in part dampens the effect of wages on the corporate earnings premium, as explored in more detail later.

Figure 2a shows that individuals with higher managerial ability tend to become entrepreneurs. As managerial ability increases, individuals tend to shift from corporate sector employment to entrepreneurship, with a slight decline in the allocation of individuals into entrepreneurial sector work across managerial ability levels. Figure 2b illustrates how individuals at a given worker productivity level are allocated across the two sectors and entrepreneurship. As worker productivity increases, the fraction of individuals who work in either sector increases, whereas the fraction of individuals who are entrepreneurs declines. Higher average worker productivity in the entrepreneurial sector stems from higher productivity workers choosing to accept employment when faced with an entrepreneurial offer, rather than waiting for a corporate sector job opportunity. The opportunity cost of unemployment for these workers is too high relative to the expected value of a better offer in the future. Notice that entrepreneurial sector workers also transition to the corporate sector through an exogenous job-to-job shock. This feature of the model also partially offsets the wage penalty for working in the entrepreneurial sector. Both figures show percents of the total population in the economy.

A striking feature of the baseline calibration is that the average assets of the workers in the corporate sectors is about 27% higher than that in the entrepreneurial sector, as seen in Table 3. A similar result is obtained when comparing medians, for which the model does a better job of matching for the case of recent hires. Figure 2c shows the distribution of worker assets by sector. The distribution in the entrepreneurial sector is more skewed, with a larger mass over the range of low asset levels. This finding suggests that workers with low assets are more likely to choose entrepreneurial sector employment, rather than waiting for a corporate sector job. However, these results are also in part due to the effect that corporate sector workers accumulate more assets during their employment spells than entrepreneurial sector workers.

To disentangle the two effects, consider the distribution of workers in their first quarter of a job. These workers are precisely the ones whose decisions reflect selection based on asset holdings around the time they choose where to work. As seen in Table 3, in the model the

average assets for such workers in the corporate sector is 29% higher than that of those in the entrepreneurial sector. This gap is larger than the total gap in average assets (27%) when all employed individuals in the two sectors are considered, not just the recently employed ones. In the data, the corresponding gaps in average assets for workers in their first quarter of job and for all employed workers are about 6% and 25%, respectively.[28] Therefore, the asset gap in the first quarter is about 20% of the gap for all levels of worker tenure in the data. In the baseline model, the entire asset gap for worker of all tenure levels is accounted for by new hires. A similar conclusion applies when comparing medians. The distribution of assets for workers in their first quarter of job is plotted in Figure 2d, which shows a similar pattern to Figure 2c. Both figures indicate that individuals with lower assets are more likely to take jobs in the entrepreneurial sector than in the corporate sector.

Is this pattern of worker sorting by assets consistent with the data? Unfortunately, there are no survey data that allow for a detailed analysis of wealth holdings by firm type, in particular, by firm age, on which the definition of entrepreneurial firms is based. Section 8 uses a novel combination of administrative records and survey data to confirm the presence of asset differentials by firm age, even after controlling for potentially confounding observables, such as worker demographics and industry composition. As the results in Section 8 suggest, the worker asset differential across the two sectors in the model is broadly consistent with the sorting by assets observed in the data, even though the asset ratios are not explicitly targeted in the calibration of the model.

The model also embeds mechanisms that generate earnings and wealth inequality. While high-wage corporate sector employment and entrepreneurship help individuals accumulate wealth at a higher rate, low-wage entrepreneurial sector employment and nonemployment can slow down such accumulation. The availability of jobs in the entrepreneurial sector nevertheless allows individuals who would otherwise be nonemployed to generate wealth, compared to an environment where there is no entrepreneurial sector. Additionally, the option to become an entrepreneur itself enables some individuals to reach the right tail of the wealth distribution. Wealth or earnings inequality moments are not targeted as part of the baseline calibration. However, one can get a sense of how well the model performs in these dimensions by comparing the shares of wealth and earnings in different quintiles of the respective distributions to those obtained from the Panel Study of Income Dynamics (PSID) (see An et al. (2009)).

Table 4 indicates that the model does fairly well in capturing the observed distributions of earnings and wealth for the first four quintiles, though less so for the top quintile's share of total wealth in the economy – 69% in the model versus 76% in the data. A model that reflects taxes and transfer programs might better capture the amount of wealth held by each wealth quintile. A

[28] See Section 8 on the details of the calculation of asset values for individuals based on data.

life-cycle retirement motive can also improve model fit by providing an incentive for the highest wealth quintile to hold more wealth in anticipation of retirement. These mechanisms have been shown to facilitate the replication of wealth distribution in models (see, e.g., Alonso and Rogerson (2010) and Kopecky and Koreshkova (2014)). The model's performance regarding inequality is similar to more standard models of a heterogenous agent economy (e.g., An et al. (2009)), rather than the models of entrepreneurship such as Cagetti and De Nardi (2006) or Buera and Shin (2011), where calibration explicitly targets the moments of the wealth distribution.

If entrepreneurial firms pay lower wages per efficiency unit, why does anyone work for them at all? The answer lies in the patterns exhibited by the average assets ratios in the two sectors. Because nonemployed individuals with low assets are not wealthy enough to secure a smooth stream of consumption while unemployed, they cannot afford to reject a job offer from the entrepreneurial sector and wait for a job offer from the corporate sector. In other words, the opportunity cost of waiting for a corporate offer is high for these individuals. Therefore, they more readily take entrepreneurial job offers.

In the model, individuals start with zero assets and asset accumulation takes time. Therefore, younger individuals have, on average, lower assets compared with older individuals. An implication is that younger individuals are also more likely to take jobs in the entrepreneurial sector. This implication is consistent with the finding that younger individuals tend to work more for younger firms (see Ouimet and Zarutskie (2014)).

Worker sorting has implications on earnings and wealth inequality. On the one hand, the presence of the entrepreneurial sector allows nonemployed, low-wealth individuals to build assets. On the other hand, the wage and earnings differentials between the two sectors leads to a faster accumulation of wealth for workers in the corporate sector, who are already more wealthy on average when they take corporate sector jobs. While the former effect works to reduce wealth inequality, the latter can propagate it. Because worker sorting based on assets and productivity is an important implication of the model, the next section explores in further detail the model's mechanisms that generate worker sorting.

6 An Analysis of the Model's Key Features

This section explores how key features of the model drive the nature of worker sorting and equilibrium allocations across the two sectors. Worker sorting is a result of the model's assumptions on financial and labor market frictions, and production technology, which together generate different labor prices for the entrepreneurial and corporate sectors. Since workers differ by assets and productivity, they have different incentives to choose work in one sector versus the other, given different labor prices and frictions. At the same time, the technology operated by

the entrepreneurial sector and the timing of entrepreneurial ability shocks determine the scale of an entrepreneurial firm along with the presence of borrowing constraints. To disentangle and isolate various factors that determine the degree of sorting in the model, key features of the model are altered one by one, and their effects on sorting are studied.

6.1 Labor Prices

The model is distinguished by the presence of two prices for labor, unlike most models of entrepreneurship which feature a uniform price. The presence of differential wages by sector is critical for capturing worker incentives for sorting. To analyze this connection further, two approaches are considered. In the first approach, the value of the entrepreneurial wage is forced to be equal to that of the corporate sector wage in baseline equilibrium. The remaining prices and all other model parameters, including search frictions, are held at their baseline values. This approach explores the effects of raising the entrepreneurial sector wage to the corporate level without allowing the rest of the model parameters to respond to this change.

The second approach is to let the corporate sector set a uniform wage for the entire economy, and to do the calibration exercise all over again to recover a new set of values for the parameters that match the data targets under this uniform price – instead of leaving the parameters fixed at their baseline values as in the first exercise. One obvious drawback of both of these approaches is that the wage set by the corporate sector does not necessarily clear the labor market for the entrepreneurial sector. In addition, the first approach does not result in market clearing for capital or the corporate sector labor market. Therefore, both the first and second approaches are necessarily partial-equilibrium exercises.

Table 5 compares the results of the first approach with the baseline. The higher wage in the entrepreneurial sector virtually eliminates any of the corporate earnings premium observed in the baseline economy. The entrepreneurship rate decreases from 4.0% found in the baseline economy to 0.9%. Only those with high levels of ability choose to become entrepreneurs. The entrepreneurial employment share increases to 15.5%. Sorting based on worker productivity is virtually eliminated, and the direction of worker sorting by assets is reversed, though sorting is now much weaker. This large change in allocations illustrates that relatively low wage in the entrepreneurial sector in the baseline model plays a significant role in reducing the incentives for workers to take jobs in this sector.

Next, the calibration procedure is allowed to search over all parameters, with the restriction of equal labor prices across the two sectors. The wage rate is set by the corporate sector and is taken as given by the entrepreneurial sector. Table 6 summarizes the results of this approach. While the new calibration is able to match most of the targeted moments, the model is unable to replicate

the observed corporate earnings premium. There are no differences in average productivity of workers across sectors. This suggests that differential wages are key in capturing the observed earnings differences between workers in this economy.

The above partial equilibrium exercises illustrate that wage differential plays an important role in generating worker sorting. Since wage rates are endogenous in this framework, it is possible to explore which features of the model generate sorting through equilibrium wage changes. The following sections do so by altering some key features of the model and allowing the labor prices to adjust to their equilibrium values. These exercises turn out to be critical in identifying the degree of worker sorting that can be attributed to each feature.

6.2 Labor Market Frictions

In the baseline model, most job offers come from the corporate sector ($\gamma_f > \gamma_e$) and separation rate is larger in the corporate sector ($\phi_f > \phi_e$). To evaluate the extent these labor market frictions account for worker sorting, labor market frictions are now equalized across sectors, holding all other parameters fixed at their baseline values. In particular, a stationary equilibrium is found with equal job finding probabilities by sector ($\gamma_e = \gamma_f = 0.27$), as well as equal separation rates ($\phi_e = \phi_f = 0.10$). These parameter choices keep the overall job finding rate at the baseline model level, $\gamma_e + \gamma_f = 0.54$ and allow for exogenous shocks that are consistent with gross flows documented in Krusell et al. (2011). The remaining parameter values are kept at their baseline values, and the prices for labor are allowed to adjust to clear the labor market.

The results from this experiment are in the column labelled "Frictions" in Table 7. The key finding is that removal of the differences in labor market frictions has an important effect on the degree of sorting. Employment share of the entrepreneurial sector increases substantially from 14.0% to 47.4%, while the average asset ratio decreases from 1.27 to 1.07. The ratio of average labor productivity increases from 0.95 to 0.97. Overall, worker sorting based on assets becomes substantially weaker. The corporate sector wage premium prevails.

Sector-specific labor market frictions are critical in generating observed worker allocations across sectors. Equalizing frictions reduces sorting by assets more substantially than the other experiments in this section. It also allocates half of total employment to the entrepreneurial sector despite a substantial wage premium, suggesting that technology underlying entrepreneurial production is important in generating earnings and wage differentials between sectors. These technologies were calibrated to be quite different, but the addition of uncertainty in entrepreneurial businesses can also be important. The role of uncertainty is explored in the following section.

6.3 Entrepreneurial Uncertainty

The previous section has demonstrated that when labor market frictions are the same across sectors, the sectoral difference in wage rate still arises, owing to the differences in technology and borrowing constraints. The technology of entrepreneurial firms is defined by the entrepreneurial ability process and the parameters of the production function. In the baseline model, the timing of the resolution of the entrepreneurial uncertainty is such that the entrepreneurial ability draw is realized *after* the decision to become an entrepreneur is made. How much does this uncertainty matter for worker sorting and equilibrium allocations?

The column labelled "Uncertainty" in Table 7 shows allocations and prices for a stationary equilibrium where entrepreneurial ability shocks are revealed *before* the occupational choice is made. All parameters are held at their baseline values. Relative to the baseline economy, the entrepreneurial sector employment share increases from 14.0% to 14.7%, and the fraction of entrepreneurs remains essentially the same. Additionally, observe that all firms in the entrepreneurial sector now operate below their optimal scale: all entrepreneurs are borrowing constrained. Relative to the economy with no differences in labor market frictions and the baseline, worker sorting based on productivity alone does not change by much. However, the effects of removing uncertainty on sorting are highly pronounced. The asset ratio in the economy with no uncertainty is 1.11, slightly larger than the value 1.07 obtained when labor market frictions are removed. Note that sorting prevails despite the large decline in the wage ratio from 1.23 to 1.12. When uncertainty is eliminated, entrepreneurial wages increase dramatically, preventing an increase in entrepreneurship rate relative to the baseline despite a large improvement in the distribution of entrepreneurial productivity (see Figure 3).[29]

6.4 Borrowing Constraint

In the baseline model, a large fraction (75%) of entrepreneurs are borrowing constrained when operating their business. The exercise in the previous section demonstrated that the removal of the uncertainty about entrepreneurial ability reduces the incidence of small entrepreneurial firms, while rendering all entrepreneurs borrowing-constrained. This finding suggests that borrowing constraints are critical in determining the scale of businesses operated by entrepreneurs, and consequently the wage rates and worker sorting.

To explore how borrowing constraints matter for worker sorting, the borrowing constraint

[29] In the sensitivity analysis section below, the model's parameters are also re-calibrated to match the calibration targets without the entrepreneurial ability uncertainty. It turns out this type of uncertainty is not critical in capturing the observed earnings premium for the corporate sector as long as entrepreneurial ability parameter can be decreased sufficiently to shift the managerial ability distribution closer to the baseline economy.

parameter, b, is raised to 2.0 from its baseline value of 1.5, keeping all other parameters fixed at their baseline values. This represents a doubling of the assets that entrepreneurs are able to borrow. The properties of the stationary equilibrium are shown in the last column of Table 7, labelled "Borrowing". Relaxing the borrowing constraint has a small effect on the distribution of employment across the two sectors. Increased borrowing provides an incentive for workers to become entrepreneurs, and this incentive is not offset by the increase in entrepreneurial sector wage rate. As a result, the share of employment in the entrepreneurial sector is approximately equal in the baseline and the equilibrium with relaxed borrowing constraints.

The ratios of average worker productivity and assets also change. In particular, the ratio of average worker productivity is now 0.97, compared with 0.95 in the baseline. Comparing average worker assets across the two sectors also indicates a reduction in worker sorting based on assets. The average asset ratio is 1.18, compared to its baseline value of 1.27. These changes in allocations are smaller in magnitude than those found in the previous experiments. The corporate earnings declines from 16.6% to 14.3%. This decline is larger than the one that occurs when frictions are equalized. The wage ratio also declines, but not to the extent in the case of no uncertainty.

The three exercises above illustrate the mechanisms that drive worker sorting in the baseline economy. The analysis suggests that labor market frictions, uncertainty in entrepreneurial ability, and borrowing constraints are important drivers of worker sorting. In particular, sector-specific labor market frictions result in large changes in allocations of labor and assets across sectors. However, there is room for exploring further how these key elements impact worker sorting. Further analysis is carried out in Section 7.

6.5 Separation Shocks

A noteworthy feature of the baseline calibration is that separation rate parameters differ across sectors. To what extent does the difference in separation rates matter? Several experiments are conducted by changing the separation rate parameters for each sector, holding all other parameters fixed at their baseline values. The results are in Table 8.

In the first experiment, the entrepreneurial sector separation rate is raised to the separation rate in the corporate sector. This is similar to the experiment in Section 6.2, except that job finding rates remain fixed at their baseline values. While the overall employment rate remains unchanged, the share of employment in the entrepreneurial sector declines from 14.0% to 11.1%. Wage rate in the entrepreneurial sector also increases, accompanied by a decline in the fraction of entrepreneurs in the economy. There is also a small decline in the corporate earnings premium.

In the second experiment, the corporate separation rate is reduced to the value of the sep-

aration rate in the entrepreneurial sector. Equilibrium allocations change in a fashion similar to the first experiment. A smaller separation probability in the corporate sector makes it more attractive for workers relative to the entrepreneurial sector. As a result, entrepreneurial sector employment decreases from 14.0% to 9.5%. Corporate earnings premium declines much more compared to the first experiment.

Overall, an increase in entrepreneurial separation rate has a similar effect on the economy as a decrease in corporate separation rates. In both cases, working in the entrepreneurial sector becomes less attractive. To clear the labor market, entrepreneurial sector wage increases relative to the baseline value, thereby reducing the corporate earnings premium and the entrepreneurship rate.

The experiments also illustrate that the model allocations are sensitive to separation rates. A reduction of the corporate sector separation rate to equal the entrepreneurial sector separation rate not only reduces the entrepreneurial employment share, but also reduces the corporate earnings premium by approximately half. While sector-specific separation rates are important for model fit, they are not critical to examining the role of worker sorting across sectors. This implies that job finding probabilities are the more important friction that determines the amount of sorting found in the baseline calibration.

7 Sensitivity Analysis

The baseline model makes several stark assumptions to facilitate the analysis of various factors at work in generating worker sorting. In this section, the model is re-calibrated to match the targets of the baseline economy using alternative assumptions about key elements of the model: entrepreneurial uncertainty and borrowing constraints. The purpose of this section is to document the ability of the model to capture key targets of the calibration with alternative assumptions. The key message of this section is that labor search friction parameters and the mean entrepreneurial ability parameter are important in obtaining allocations and prices that are comparable to the baseline economy.

7.1 Entrepreneurial Uncertainty

In the baseline model, the timing of the resolution of the entrepreneurial uncertainty is such that the entrepreneurial ability draw is realized *after* the decision to become an entrepreneur is made. Table 10 compares key features of the baseline economy with an alternative economy where individuals *know* their initial entrepreneurial ability draw at the time of the entrepreneurship decision.

The economy without uncertainty in entrepreneurial ability turns out to be in many ways similar to the baseline economy, as shown in Table 9. The employment rate and the sectoral employment shares do not differ significantly. Similarly, the wage and average earning ratios are close to the baseline, and hence, worker sorting based on assets and productivity do not change substantially. The slight decline of the asset ratio from 1.27 to 1.24 indicates that sorting based on assets is somewhat less pronounced. The entrepreneurship rate is about 3.5% compared to 4.0% in the baseline. In addition, average entrepreneurial ability is also lower. Note also that parameters for labor market frictions change as a result of the new calibration of the model. Changes in these parameters restore baseline equilibrium allocations and prices without the uncertainty in entrepreneurial ability. This finding suggests that entrepreneurial uncertainty is not the primary determinant of worker sorting in the model.

7.2 Borrowing Constraint

The baseline model sets the borrowing limit at 50% of an entrepreneur's assets ($b = 1.5$). While this value of b is common in earlier models of entrepreneurship (see, e.g., Kitao (2008) and Buera and Shin (2011)), it is certainly not the only way to specify the extent of limits to borrowing. To assess the role of the borrowing constraint further, the constraint is removed entirely to allow entrepreneurs to operate their businesses at optimal scale. The parameters of the model are estimated to match the targets in the baseline with no borrowing constraint in place.

Table 10 compares the new equilibrium with the baseline. The corporate earnings premium is now higher at 21.9% compared to 16.6% in the baseline economy. The share of employment in the entrepreneurial sector is lower – 11.0% compared to 14.0% in the baseline. The two key parameters that change significantly in the new calibration are the mean of entrepreneurial ability and the corporate job finding rate. The former declines from 0.38 to 0.27 and the latter declines from 0.46 to 0.43. These changes allow the model to match the 4.0% entrepreneurship rate. Furthermore, sorting by productivity and assets is now more pronounced compared to the baseline economy. Corporate sector workers are now less productive than their entrepreneurial counterparts. At the same time, the asset ratio increases from 1.27 and 1.78, indicating a stronger worker sorting based on assets. These results demonstrate that sorting can be obtained in an economy with no borrowing constraint for entrepreneurs as long as average entrepreneurial ability is sufficiently low.

8 Some Evidence on Worker Sorting by Assets

A key prediction of the baseline model is that the average asset holdings of workers in the entrepreneurial sector is lower than that of workers in the corporate sector. This difference emerges as a combination of two effects. First, workers holding fewer assets tend to accept job offers from the entrepreneurial sector. Second, workers in the corporate sector tend to accumulate assets at a higher rate because of higher wages in that sector. Is there empirical evidence on these two types of worker sorting based on assets? The analysis to follow indicates some support for the model's predictions.

8.1 Data

Testing the model's implications on worker sorting by assets is challenging because it demands data on both workers' assets and the types of firms they work for. However, household survey data that include information on worker assets typically do not contain information on the age of a worker's employer to identify the entrepreneurial (young) firms with which workers are affiliated. To address this shortcoming, this paper brings together two sets of data. The wealth data for workers in the Survey of Income and Program Participation (SIPP), which contains information on various assets held by individuals, is merged with the Longitudinal Employer-Household Dynamics (LEHD) data that captures employment spells, earnings, and some employer characteristics for those workers.[30]

To measure worker assets, the responses in the Asset and Liabilities Topical Module collected in the 1996, 2001, 2004 and 2008 SIPP panels are used to create a net worth variable, excluding housing equity. The topical module is only collected a few times per panel, and provides limited variation over time.[31] All panels are pooled across time for analysis. Housing equity is initially excluded to prevent the undue influence on assets of large amount of wealth accumulation that results from the appreciation in house values during the sample period. Nevertheless, the results are robust to inclusion of the housing equity. The net worth variable is calculated at the household level, and used as the primary empirical counterpart to assets in the model. The net worth is the sum of the market value of assets (except housing) owned by every member of the household minus the liabilities of household members.[32] For the analysis, the net worth variable is winsorized at the

[30] For more information on LEHD data, see Hyatt et al. (2017).

[31] The topical module is administered in Waves 3, 6, 9, 12 of 1996 Panel, Waves 3, 6, 9 of 2001 Panel, Waves 3 and 6 of 2004 Panel, and Waves 4, 7, 10 of 2008 Panel.

[32] The assets included in this definition are interest-earning assets held at financial institutions, stocks and mutual fund shares, rental property value and rental income, IRA, 401K and thrift savings plans, and values of vehicles owned. The excluded assets are equities in pension plans, the cash value of life insurance policies, home value, and the value of home furnishings and jewelry.

top 1% to reduce the effects of some likely outliers. The SIPP data also provide a set of variables that describe individuals' characteristics, including gender, race, marital status, education, and age. These variables offer a rich set of controls in testing the model's implications on assets.

The individuals in the SIPP panels are linked to the LEHD data based on unique time-invariant individual identifiers called Protected Identification Keys (PIKs). The LEHD data are the universe of quarterly wage data for available states, and the SIPP quarter is the calendar quarter in which an individual's response to the survey is recorded. The indicator of employment is taken from LEHD rather than SIPP, implying that employment among SIPP respondents is not recorded in instances when it does not correspond to employment in the administrative data. However, there are several limitations to this linking procedure. First, different states enter LEHD data at different times, so the sample is restricted to data starting one year after the inclusion of a state's data series in the LEHD.[33] This approach reduces spurious identification of new hires. Second, federal government employment is excluded from the analysis since these records are not integrated in the infrastructure files. Third, self-employed workers and business owners in businesses without unemployment insurance (UI) eligible employees are excluded from the analysis.

Critically, LEHD data provides age and size measures for employers. In addition, industry affiliation for these firms are also available. For workers holding more than one job during the relevant quarter, firm age and size pertain to the firm where worker earnings were the greatest among all jobs held in that quarter.[34] In the analysis to follow, all variables denominated in dollars are converted to 2014 constant dollars, and survey weights in SIPP are used to obtain estimates that are representative of the population subject to the data limitation noted above.

8.2 Results

Table 11 shows the mean and quasi-median of net worth for workers in the sample. To be consistent with the calibrated model, the sample is grouped into workers in entrepreneurial (young firms with 0-5 years of age) and other firms (older firms with 5+ years of age).[35] All statistics indicate a stark difference in the average asset holdings of workers in entrepreneurial firms relative to others. In particular, workers in older firms have a mean net worth that is

[33]The implication of missing states in the LEHD for the calculation of person-level employment statistics is considered by Henderson and Hyatt (2012), who also provide the years that different states enter the data.

[34]For details on how separate jobs and employment spells are identified in the LEHD data, see Hahn et al. (2018).

[35]Quasi-median is calculated as an alternative to percentiles to satisfy U.S. Census Bureau's requirements for disclosure. It is the average of the observations in the data between the 45th and the 55th percentiles. The standard error for the quasi-median is obtained using a bootstrap procedure that calculates the quasi-median on 100 different 50% samples taken from the data.

about 24% percent higher than that of workers in young firms. Based on the quasi-median, this difference is even larger – about 95%.[36] Note, however, that higher wages in old firms relative to entrepreneurial firms would imply an asset differential even when workers accept job offers randomly regardless of their asset holdings, as long as employment has some persistence. On this point, Table 11 shows that the earnings of workers in older firms are in general much higher, regardless of whether the earnings is measured by the mean or quasi-median.

Further evidence on sorting of workers based on assets at the time when they take jobs can be provided by examining the assets of recently hired workers only. For this purpose, the sample is restricted to those workers who are within their first year of employment following an unemployment spell. This subsample allows for an approximation to the asset holdings of workers in the model who have recently transitioned into employment from unemployment. The results are shown in Table 11. There is a statistically significant asset differential across workers in the two types of firms when measured by either the mean or quasi-median net worth. Therefore, the asset gap is not merely a result of the fact that working for older, more established firms allows individuals to accumulate more assets over time. Workers accepting jobs in these firms are on average wealthier to start with, in line with the model's prediction on sorting based on assets. Most results are similar if firm size is used rather than firm age to define entrepreneurial firms.[37] These results are shown in Online Appendix D.

One obvious question is how much of the observed worker sorting by assets is driven by other confounding factors, such as worker characteristics. For instance, an individual's assets are likely correlated with marital status, education, and experience. To further isolate the connection between the type of the firm an individual works for and the worker's assets, Table 12 presents several estimates based on linear models of the form

$$\widetilde{a}_{it} = \alpha d_{it} + \mathbf{x}'_{it}\beta + \varepsilon_{it}, \tag{16}$$

where $\widetilde{a}_{it}$ is the inverse hyperbolic sine transformation of the net worth a_{it} for individual i in year t

$$\widetilde{a}_{it} = \ln(a_{it} + \sqrt{a_{it}^2 + 1}). \tag{17}$$

The transformation in (17) is used for at least two reasons. First, the net worth variable a_{it} has a highly-skewed, non-normal distribution. Second, there are some zero and negative values for a_{it}, which lead to omitted observations when the standard log transformation is used. On the right hand side of specification (16), d_{it} is a dummy variable indicating employment at an entrepreneurial firm (0-5 years of age), $\mathbf{x}_{it}$ is a vector of controls (including a constant),

[36] In addition, the gap between the net worth of workers generally persists at the first and third quartiles, indicating that worker sorting prevails at different parts of the asset distribution.

[37] One exception is the case of mean net worth for recent hires.

and ε_{it} is an error term. The controls $\mathbf{x}_{it}$ include the individual's gender, race, marital status, education level, age, and age-squared, as well as industry (2-digit NAICS sector), state, and year fixed effects. The model in (16) is estimated using OLS for mean effects. In addition, median regression is used to assess the effects at the median of the net worth distribution. The primary coefficient of interest is α, which measures the connection between net worth and the entrepreneurial firm category.

Table 12 indicates that the young firm category is significantly negatively associated with net worth, even after controlling for a large set of observables. This result holds for both the mean (OLS) and median regressions. More importantly, the net worth is also lower for recent hires by young firms. The baseline model is consistent with this finding. Note also that the results are generally stronger for all workers compared to recent hires. This finding supports the prediction of the baseline model that assets differential is higher, relative to recent hires, for workers who have been employed for at least a quarter. The estimates reveal that a newly hired worker in a 0-5 year old firm has approximately 18% less net worth than a worker in an old firm, holding all else fixed.[38] The gap is much larger (38%) for all workers. The regression analysis also indicates that earnings are lower for workers employed in young firms, regardless of whether they are recently hired or more tenured.

Overall, the estimates in Table 12 support the model's key finding of worker sorting based on assets. Not only the workers in entrepreneurial (younger) firms tend to have lower net worth, but also the recent hires in these firms have on average fewer asset holdings compared to their counterparts in older firms. Furthermore, the regression results are not very different, qualitatively speaking, if small firms, instead of young firms, are used as the empirical-counterpart of entrepreneurial firms – see Online Appendix D.[39]

9 Conclusion

Entrepreneurial firms, often thought of as new or young businesses, on average hire younger and less educated workers, and provide them lower earnings compared to more established firms. To understand what kind of workers match with entrepreneurial firms versus others, this paper proposed a dynamic model of entrepreneurship, which features two sectors, entrepreneurial and

[38]Note that (16) implies that the percent change in $\widetilde{a}_{it}$ going from $d_{it} = 0$ to $d_{it} = 1$ can be estimated as $100\left(\exp(\widehat{\alpha}) - 1\right)$. For values of a_{it} that are not small, $\ln \widetilde{a}_{it} \simeq \ln(2a_{it}) = \ln 2 + \ln a_{it}$, and the percent change in $(a_{it} + \sqrt{a_{it}^2 + 1})$ approximates the percent change in a_{it}.

[39]Online Appendix D also contains estimates for worker net worth by firm age and firm size that control for finer industry groups at the 4-digit SIC level. The results hold for the all worker sample, but not in the sample of new hires. This is not surprising since the new hires sample is much smaller than the all workers sample and the large number of additional fixed effects greatly reduces the precision of the estimates.

corporate, that differ in labor market frictions and prices of labor. These elements distinguish the framework from that of many existing models of entrepreneurship, which typically features a single price of labor and a unified labor market. The two sectors also possess different production technologies and face different financial constraints. These differences together lead to a divergence in sectoral wages per unit of worker efficiency and induces sorting of workers across the two sectors based on both productivity and wealth.

The calibrated model's equilibrium offers an answer to the main question of who works for whom. Among individuals who look for work, less wealthy ones tend to take up job offers from the low-paying entrepreneurial sector, instead of waiting for a corporate job offer. This tendency results in a sorting of individuals across the two sectors by assets. More productive workers are less likely to turn down an entrepreneurial job offer resulting in slightly higher average labor productivity in the entrepreneurial sector. The model is also able to account for the observed differences across the two sectors in employment shares, average worker earnings, and gross worker flows. The model's key prediction on worker sorting based on assets finds some support in the data. The workers employed in young or small firms and those workers who were recently hired by these firms possess, on average, fewer assets than their counterparts in more established firms.

Analysis of the model reveals that the differences in the price of labor and labor market frictions both play an important role in the model's ability to replicate key features of the entrepreneurial sector and to generate worker sorting. In addition, worker sorting emerges as a robust feature of the model when alternative specifications of the model's key elements are considered. The model also provides a rich framework to study the effects of various search frictions and financial constraints on the entrepreneurial sector and its workers. One avenue for future work is to assess the effect of changes in these frictions and constraints on the long-run decline of entrepreneurship in the United States, and the implications of this decline on income inequality, and the earnings and wealth of workers who tend to take jobs in entrepreneurial firms.

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Table 1. The fraction of entrepreneurs in the U.S. economy in 2000

Basis	**Fraction of entrep.**	**Non-entrep. Pay Premium**	**Employment Share**	**Source**
Young and small firms (0-5 yr & emp $\leq$ 7)	1.7%	20.8%	3.6%	LBD
Young and small firms (0-5 yr & emp $\leq$ 15)	2.0%	18.5%	5.9%	LBD
Young firms (0-5 yr)	2.2%	17.2%	15.7%	LBD
Young (0-5 yr) + small old (6+ yr & emp $\leq$ 7) firms	4.9%	39.7%	20.8%	LBD
Young (0-5 yr) + small old (6+ yr & emp $\leq$ 15) firms	5.5%	44.7%	25.4%	LBD
Small firms (emp $\leq$ 10)	4.9%	33.5%	11.8%	LBD
Small firms (emp $\leq$ 20)	5.5%	36.7%	18.6%	LBD
Small firms (emp $\leq$ 25)	5.7%	37.4%	21.0%	LBD
Young firms (0-10 yr)	3.2%	16.6%	24.8%	LBD
Young (0-10 yr) + small old (11+ yr & emp $\leq$ 20) firms	5.8%	49.8%	33.1%	LBD
Firms classified with certainty as non-public	6.0%	45.2%	44.0%	SBO
Business owners with employees (Males 25-64)	2.9%	NA	NA	SIPP
Business owners with employees (Males 25-54)	2.8%	NA	NA	SIPP
Business owners with employees (All)	2.4%	NA	NA	SIPP
Business owners with employees (All 25-54)	2.3%	NA	NA	SIPP

Notes: The data sources are Longitudinal Business Database (LBD), Survey of Business Owners (SBO), and Survey of Income and Program Participation (SIPP). Estimates pertain to the year 2000. The denominator used to calculate fraction of entrepreneurs is the population of males aged 15-64 years, unless indicated otherwise. The calculations assume that each entrepreneurial firm is owned by a single entrepreneur.

Table 2. The parameter values in the baseline model

Selected using a priori information or estimated using data

Parameter	**Value**	**Source**
Discount rate, β	0.99	Business cycle literature (Annual int. rate = 0.04)
Capital share in production, ν	0.36	Business cycle literature
Capital depreciation rate, δ	0.015	Business cycle literature (Annual rate = 0.06)
Productivity of the corporate sector, A	0.36	Normalization ($A = e^{-1}$)
Labor productivity, $\{\rho_z, \sigma_z\}$	$\{0.97, 0.13\}$	Heathcote et al. (2010)
Entrepreneurial ability (Persistence), $\{\rho_\theta, \sigma_\theta\}$	$\{0.30, 0.18\}$	Estimated based on Abraham and White (2015)
Returns-to-scale in entrepreneurship, ξ	0.88	Estimated based on Abraham and White (2015)
Entrepreneur transition rate into corporate sector, p	0.001	IPO and Merger and Acquisition rates

Recovered using the calibration procedure

Parameter	**Value**	**Target(s)**
Disutility from labor, α	0.37	Employment-to-population ratio (80%)
Job offer rate, $\{\gamma_e, \gamma_f\}$	$\{0.07, 0.46\}$	N-to-E rate, employment shares
Job-to-job transition rate parameter, $\bar{\gamma}$	0.10	Job-to-job transition rate
Job separation rates, $\{\phi_e, \phi_f\}$	$\{0.05, 0.10\}$	E-to-N rate, earnings premium
Entrepreneurial ability (Mean), $e^{-\mu}$	0.38	Fraction of entrepreneurs (4.0%)

Notes: See Online Appendix B for the estimation of returns-to-scale for entrepreneurs and the parameters for the entrepreneurial ability process. Job separation, job finding rates, and job-to-job transition rates are taken from LEHD data. The fraction of entrepreneurs is based on the estimates in Table 1.

Table 3. The properties of the baseline model

Variable	**Model**	**Data**
Targeted Moments		
Employment-to-population ratio	82.5%	80.0%
Share of employment (Entrepreneurial sector)	14.0%	16.0%
Fraction of entrepreneurs	4.0%	4.0%
Corporate average earnings premium	16.6%	17%
Employment-to-nonemployment (E-to-N) flows	10.3%	8.5%
Nonemployment-to-employment (N-to-E) flows	39.0%	34.0%
Job-to-job flows	1.9%	1.6%
Untargeted Moments		
Ratio of average entrepreneur-to-worker assets (Entrepreneurs/Workers)	1.74	1.72
Ratio of average worker productivity (Corporate/Entrepreneurial)	0.95	NA
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.25
Ratio of median worker assets (Corporate/Entrepreneurial)	1.34	1.95
Ratio of average worker assets in first quarter of job (Corporate/Entrepreneurial)	1.29	1.06
Ratio of median worker assets in first quarter of job (Corporate/Entrepreneurial)	1.33	1.36
Wage ratio (w_f/w_e)	1.23	NA

Notes: Employment-to-population ratio is based on working age males (ages 15-64). Share of employment in the entrepreneurial sector and corporate earnings premium are based on the Longitudinal Business Database (LBD). Fraction of entrepreneurs is based on the estimates in Table 1. The estimates for average and median worker assets are based on Survey of Income and Program Participation (SIPP)–see Section 8. E-to-N, N-to-E, and job-to-job flows are taken from LEHD data.

Table 4. Inequality in the baseline model

	Share of Wealth		**Share of Earnings**	
Quintile	**Model**	**Data**	**Model**	**Data**
1st	0.8%	−0.5%	3.3%	7.5%
2nd	3.0%	0.5%	12.3%	11.3%
3rd	7.7%	5.0%	15.1%	18.7%
4th	18.9%	18.7%	17.3%	24.2%
5th	69.5%	76.2%	52.1%	38.2%

Notes: Wealth is measured by assets. The only negative value in the table is due to the high prevalence of negative net assets at the bottom of the wealth distribution.

Table 5. The role of wages in worker sorting and equilibrium allocations

Variable	**Baseline**	**Equal Wages** ($w_e = w_f$)
Employment-to-population ratio	82.5%	83.3%
Share of employment (Entrepreneurial sector)	14.0%	15.5%
Fraction of entrepreneurs	4.0%	0.9%
Corporate average earnings premium	16.6%	0.0%
Wage ratio (w_f/w_e)	1.23	1.00
Employment-to-nonemployment (E-to-N) flows	10.3%	10.0%
Nonemployment-to-employment (N-to-E) flows	39.0%	46.8%
Job-to-job flows	1.9%	1.9%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	1.00
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	0.97

Notes: The column "Equal Wages" pertains to the case where the entrepreneurial sector wage is set to be equal to the corporate sector wage in the baseline. All other parameters are fixed at their baseline values.

Table 6. Parameters under wage setting by the corporate sector

Parameter	**Baseline**	**Wage Setting by Corporate Sector**
Employment-to-population ratio	82.5%	83.1%
Share of employment (Entrepreneurial sector)	14.0%	15.5%
Fraction of entrepreneurs	4.0%	1.0%
Corporate average earnings premium	16.6%	0.0%
Wage ratio	1.23	1.00
Employment-to-nonemployment (E-to-N) flows	10.3%	10.0%
Nonemployment-to-employment (N-to-E) flows	39.0%	46.1%
Job-to-job flows	1.9%	2.1%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	1.00
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	0.98

Notes: The model where wage is set by the corporate sector re-estimates all model parameters to match the data targets.

Table 7. Decomposing sources of worker sorting in baseline model

Variable	**Baseline**	**Frictions**	**Uncertainty**	**Borrowing**
Employment-to-population ratio	82.5%	80.4%	82.4%	82.4%
Share of employment (Entrepreneurial sector)	14.0%	47.4%	14.7%	14.2%
Fraction of entrepreneurs	4.0%	12.2%	4.1%	4.9%
Share of entrepreneurs with binding borrowing constraint	0.75	0.49	1.00	0.74
Ratio of average worker productivity (Corporate/Entrep.)	0.95	0.97	0.98	0.97
Corporate average earnings premium	16.6%	16.2%	10.6%	14.3%
Wage ratio (w_f/w_e)	1.23	1.20	1.12	1.18
Ratio of average worker assets (Corporate/Entrep.)	1.27	1.07	1.11	1.18
Ratio of average worker assets in first quarter of job (Corporate/Entrep.)	1.29	1.07	1.12	1.19

Notes: The column "Frictions" refers to the model where job finding rates and separation rates are equal across sectors.

The column "Uncertainty" refers to the model where the uncertainty about the initial draw of entrepreneurial ability is removed at the time entrepreneurship decision is made. The column "Borrowing" refers to the model where the borrowing limit is increased to 2.0 for entrepreneurs.

Table 8. The role of separation rates

Variable	**Baseline**	**Corp Sep. Rates** ($\phi_e = \phi_f$)	**Ent. Sep. Rates** ($\phi_f = \phi_e$)
Employment-to-population ratio	82.5%	81.9%	87.4%
Share of employment (Entrepreneurial sector)	14.0%	11.1%	9.5%
Fraction of entrepreneurs	4.0%	3.7%	3.2%
Corporate average earnings premium	16.6%	16.5%	6.8%
Wage ratio	1.23	1.21	1.21
Employment-to-nonemployment (E-to-N) flows	10.3%	11.4%	6.8%
Nonemployment-to-employment (N-to-E) flows	39.0%	40.6%	36.5%
Job-to-job flows	1.9%	1.7%	1.6%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	0.96	0.90
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.24	1.36

Notes: Each column represents equilibrium allocations that vary from the baseline by probability of separation to nonemployment. See text for details.

Table 9. The properties of the model with no entrepreneurial uncertainty

Variable	**Baseline**	**No Uncertainty**
Employment-to-population ratio	82.5%	81.1%
Share of employment (Entrepreneurial sector)	14.0%	13.7%
Fraction of entrepreneurs	4.0%	3.5%
Corporate average earnings premium	16.6%	15.6%
Wage ratio (w_f/w_e)	1.23	1.22
Employment-to-nonemployment (E-to-N) flows	10.3%	11.1%
Nonemployment-to-employment (N-to-E) flows	39.0%	38.4%
Job-to-job flows	1.9%	1.8%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	0.95
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.24
Disutility from labor, α	0.37	0.38
Entrepreneurial sector job offer rate, γ_e	0.07	0.06
Corporate sector job offer rate, γ_f	0.46	0.43
Job-to-job transition rate parameter, $\bar{\gamma}$	0.10	0.09
Entrepreneurial sector job separation rate, ϕ_e	0.05	0.06
Corporate sector job separation rate, ϕ_f	0.10	0.09
Entrepreneurial ability (Mean), $e^{-\mu}$	0.38	0.36

Notes: The column "No Uncertainty" refers to the model where the uncertainty about the initial draw of entrepreneurial ability is removed at the time entrepreneurship decision is made. All model parameters are re-estimated to match the data targets.

Table 10. The properties of the model with no borrowing constraint

Variable	**Baseline** ($b = 1.5$)	**No Borrowing Limit** ($b = \infty$)
Employment-to-population ratio	82.5%	78.7%
Share of employment (Entrepreneurial sector)	14.0%	11.0%
Fraction of entrepreneurs	4.0%	2.6%
Corporate average earnings premium	16.6%	21.9%
Wage ratio (w_f/w_e)	1.23	1.39
Employment-to-nonemployment (E-to-N) flows	10.3%	12.0%
Nonemployment-to-employment (N-to-E) flows	39.0%	39.0%
Job-to-job flows	1.9%	1.9%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	0.88
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.78
Disutility from labor, α	0.37	0.36
Entrepreneurial sector job offer rate, γ_e	0.07	0.02
Corporate sector job offer rate, γ_f	0.46	0.43
Job-to-job transition rate parameter, $\bar{\gamma}$	0.10	0.12
Entrepreneurial sector job separation rate, ϕ_e	0.05	0.06
Corporate sector job separation rate, ϕ_f	0.10	0.11
Entrepreneurial ability (Mean), $e^{-\mu}$	0.38	0.27

Notes: In the model with no borrowing limit, all model parameters are re-estimated to match the data targets.

Table 11. Household net worth by firm age

	Mean		**Quasi-median**	
Firm Age (years):	**0-5**	**6+**	**0-5**	**6+**
Net worth (All workers)	$101,200	$126,100	$11,970	$23,360
s.e.	(1,690)	(607)	(267)	(201)
N	23,000	205,000	23,000	205,000
Net worth (Recent hire)	$90,720	$96,290	$8,490	$11,550
s.e.	(2,373)	(1,068)	(337)	(203)
N	11,000	55,000	11,000	55,000
Earnings (All workers)	$10,230	$12,940	$7,364	$9,936
s.e.	(167)	(289)	(47)	(221)
N	23,000	205,000	23,000	205,000
Earnings (Recent hire)	$7,534	$10,010	$5,200	$6,316
s.e.	(94)	(1,060)	(81)	(40)
N	11,000	55,000	11,000	55,000

Notes: Standard errors in parentheses. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. For quasi-median the standard error is calculated using bootstrap.

Table 12. Regression analysis of household net worth and earnings by firm age

	Estimated Young Firm Coefficient	
Dependent Variable	**OLS**	**Median**
Net worth (All workers)	-0.480^{***}	-0.240^{***}
s.e.	(0.079)	(0.028)
N	228,000	228,000
Net worth (Recent hire)	-0.196^{*}	-0.100^{**}
s.e.	(0.116)	(0.042)
N	66,000	66,000
Earnings (All workers)	-0.297^{***}	-0.115^{***}
s.e.	(0.023)	(0.006)
N	228,000	228,000
Earnings (Recent hire)	-0.225^{***}	-0.105^{***}
s.e.	(0.040)	(0.010)
N	66,000	66,000

Notes: Standard errors in parentheses (clustered by time and industry for OLS). All regressions include gender, race, marital status, education level dummies, age, and age-squared, as well as state, industry (2-digit NAICS) and year fixed effects. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. (*), (**), and (***) indicate statistical significance at 10, 5, and 1% levels, respectively.

Figure 1. The distributions of entrepreneurial ability, assets, capital input and labor input – baseline model

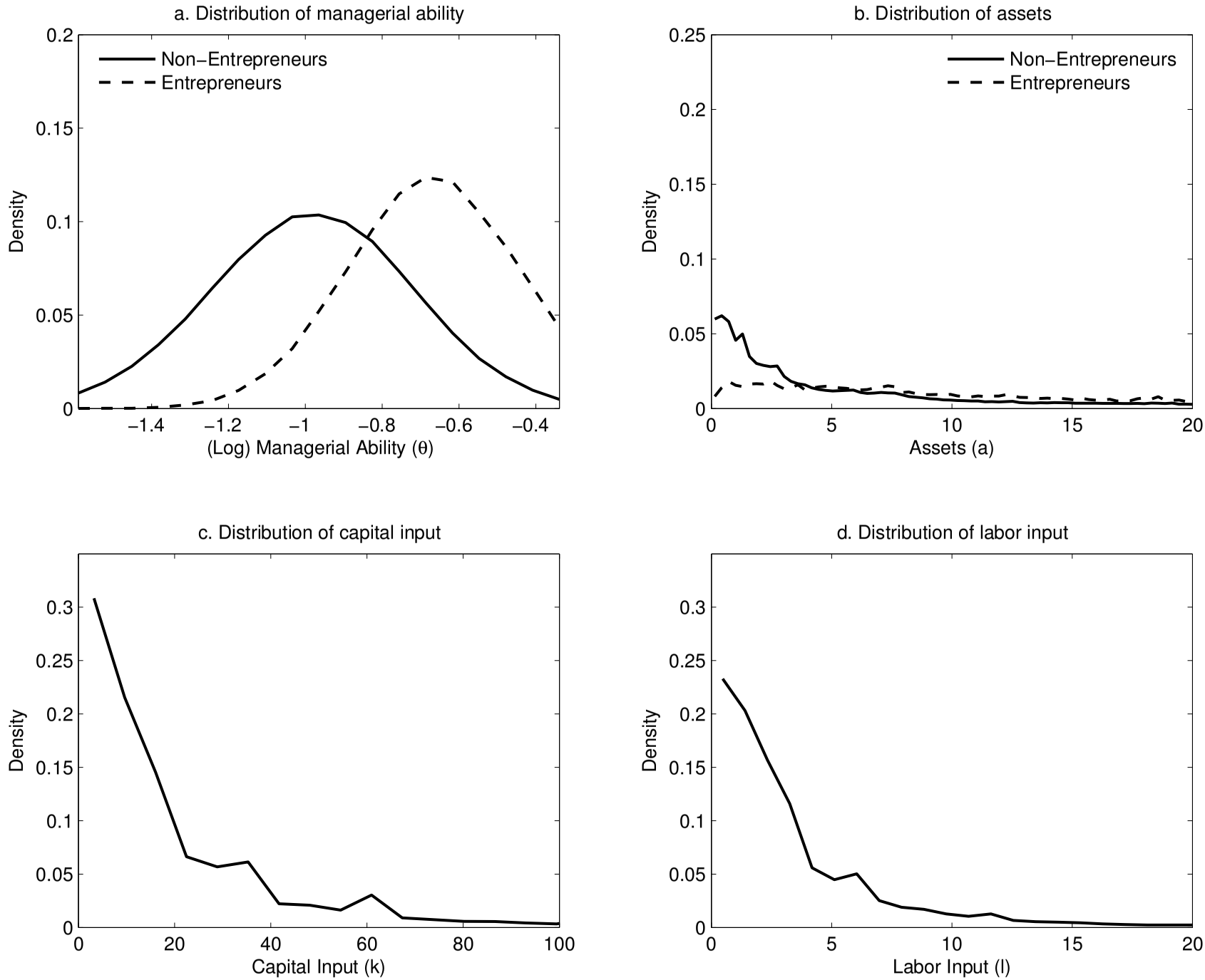


Figure 2. The allocation of individuals – baseline model

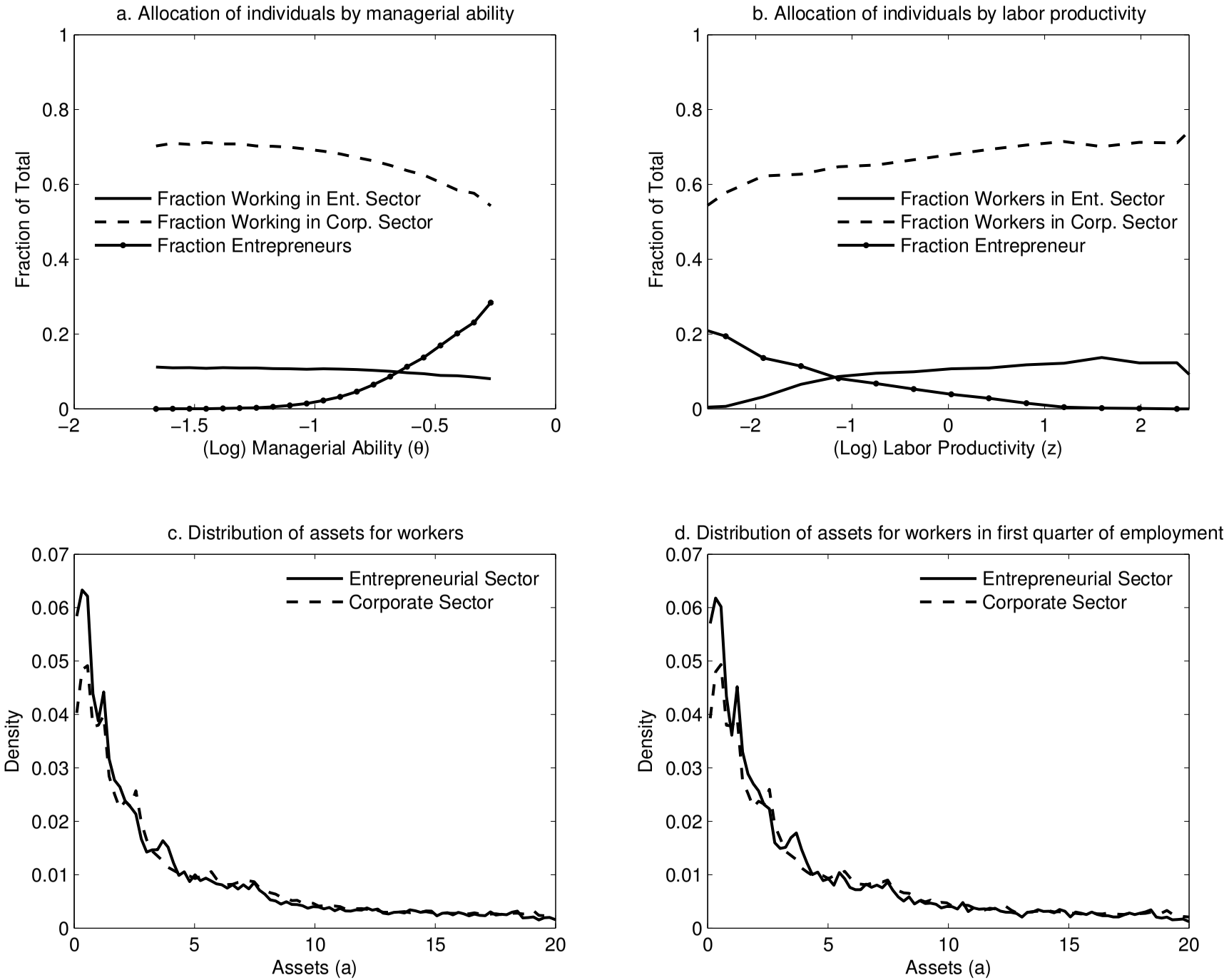
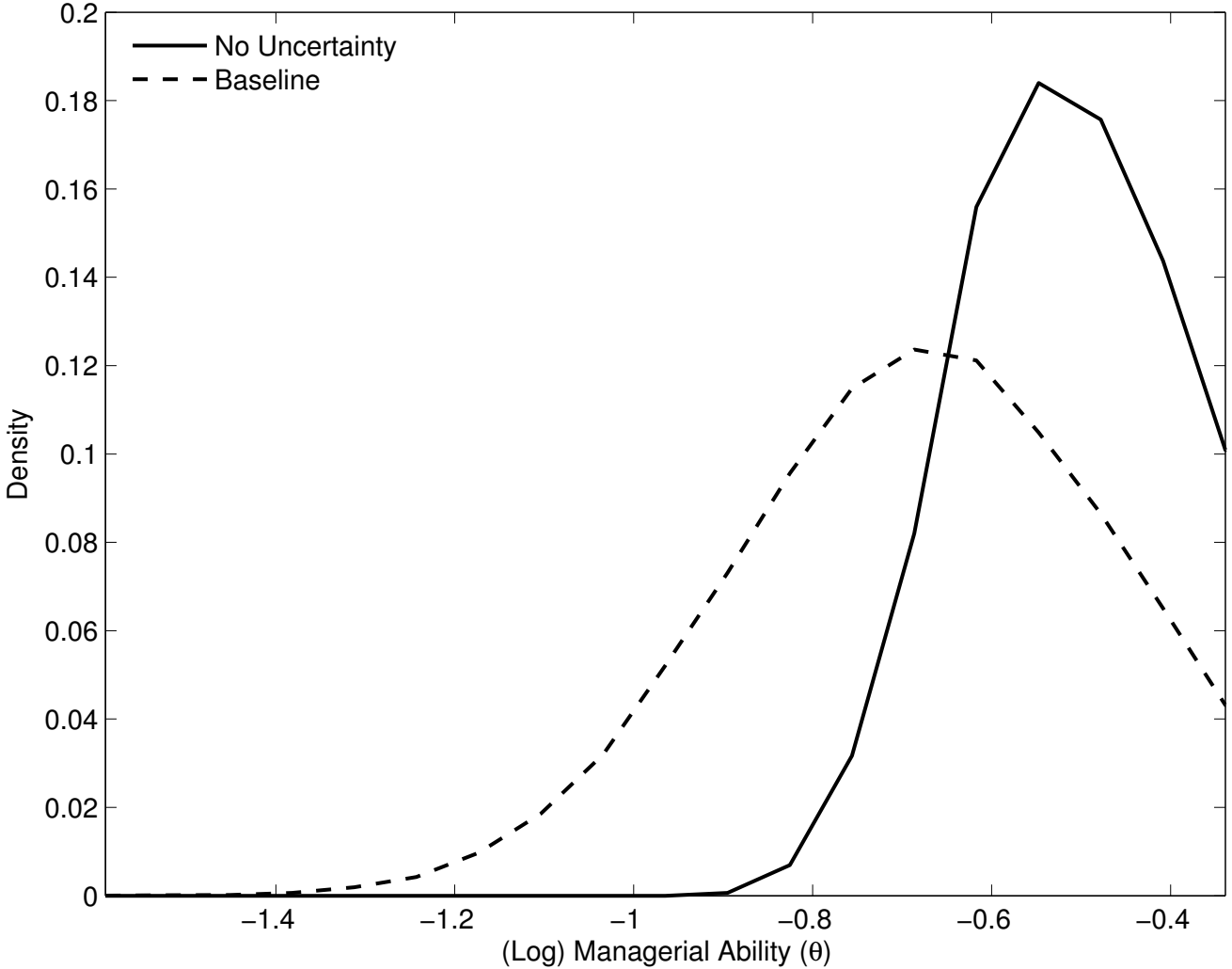


Figure 3: The Distribution of managerial ability among entrepreneurs



Online Appendix for "Who Works for Whom? Worker Sorting in a Model of Entrepreneurship with Heterogenous Labor Markets"*

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A Algorithm for Solving The Model's Equilibrium

A stationary equilibrium of the model is computed using an algorithm based on Huggett and Ventura (1999). The algorithm finds an equilibrium by iterating over value functions and decision rules over a discretized state space. Labor productivity and entrepreneurial ability processes are discretized on a 21-point support for the distribution implied by the process. The support is bounded below and above the mean by 2.5 times the standard deviation. The asset grid is discretized to 201 points with linear interpolation of all functions between grid points. The spacing between points on the asset grid increases with asset levels. Asset grid points are placed according to $a_1 = 0$, $a_j = \psi j^{\chi}$ for $j = 2, ..., 201$, where $\chi = 2.0$, $\psi = \bar{a}/(201^{\chi})$ and $\bar{a}$ is an upper bound. The algorithm is as follows.

1. Guess a value for the capital-labor ratio in the corporate sector, K/L, and δ_τ,
2. Calculate the values $w_f = (1-\nu)AK^{\nu}L^{-\nu}$ and $r = \nu AK^{\nu-1}L^{1-\nu} - \delta - \delta_\tau$,
3. Set the initial value for the entrepreneurial sector wage equal to the corporate sector wage: $w_e = w_f$,

*Any opinions and conclusions expressed herein are those of the authors and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential data are disclosed.

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4. Calculate the optimal decision rules $c^i(s)$, $a^{i\prime}(s)$, $h^d(\widetilde{s})$, $m^d(\widetilde{s})$, $k(s), l(s)$, $(i, d \in \{n, f, e, m\})$

5. Calculate K'/L', $\int l(s)d\Psi^m(s)$, $\int p\tau(s)d\Psi^e(s)$, and $\int zd\Psi^e(s)$ implied by the optimal decision rules,

6. If

$$\begin{aligned} |K'/L' - K/L| &< \eta_1, \\ |\int l(s)d\Psi^m(s) - \int zd\Psi^e(s)| &< \eta_2, \\ \left|\int p\tau(s)d\Psi^e(s) - \delta_\tau K\right| &< \eta_3 \end{aligned}$$

for small $\eta_1, \eta_2, \eta_3 > 0$, then a stationary equilibrium has been found. Otherwise, update K/L, $\{w_e, w_f\}$, and δ_τ, and repeat steps 4-6.

The parameter vector $M = \{\alpha, \gamma_e, \gamma_f, \bar{\gamma}, \phi_e, \phi_f, \mu\}$ is recovered using a Nelder-Mead Simplex algorithm, where the objective function is set to minimize the distance between the target and simulation moments. Target moments m_j $(j = 1, ..., J)$ are described in Section 4. The objective function used is

$$\sum_{j=1}^{J}[(\ln(m_j/d_j))^2],$$

where m_j is the moment j calculated using simulated data in the stationary equilibrium and d_j is the corresponding data moment. The Fortran 90 code for the simplex algorithm is taken from the public domain and was written by Alan Miller.[1]

B Estimation of the Parameters $\rho_\theta, \sigma_\theta$, and ξ

The estimation of the decreasing returns parameter, ξ, for entrepreneurial firms, and the parameters for the entrepreneurial productivity process, $\{\rho_\theta, \sigma_\theta\}$, is based on the framework of Abraham and White (2015).[2] The framework allows the estimation of the parameters $\{\rho_\theta, \sigma_\theta, \xi\}$ simultaneously. Consider a production function for a manufacturing firm i in the form of

$$y_{it} = \theta_{it}\left(k_{it}^{a_k} l_{it}^{a_l} x_{it}^{1-a_k-a_l}\right)^{\xi}, \tag{1}$$

which includes materials and energy, x_{it}, as an input, and a productivity process $\ln\theta_{it} = (1 - \rho_\theta)\mu_i + \delta_t + \rho_\theta \ln\theta_{it-1} + \epsilon_t$, where μ_i is a firm-specific productivity parameter, δ_t is a year effect that captures general changes in productivity that apply to all firms, and $\epsilon_t \sim N(0, \sigma_\theta)$. The

[1] Available at `http://jblevins.org/mirror/amiller/`.

[2] Also see Castiglionesi and Ornaghi (2013) for a similar estimation methodology.

parameters ρ_θ and σ_θ are allowed to vary across industries. The inclusion of the materials and energy in the production function controls for the use of intermediate inputs (materials and energy) in estimating the underlying total factor productivity process. The estimation also allows for a markup, η, common to all firms in an industry, which can be thought of as the average markup across firms that is assumed to be constant over time. Abraham and White (2015) estimate the parameters, ξ, ρ_θ and σ_θ in a GMM framework using the log-linear form of the production function and the Solow residual obtained from the gross output and cost shares of the inputs. See Abraham and White (2015) or Castiglionesi and Ornaghi (2013) for a derivation of the exact model estimated.

The data used for the estimation is the U.S. Census Bureau's Annual Survey of Manufactures (ASM), which provides an unbalanced panel of manufacturing establishments for the period 1972-2009. The data include, for each establishment, annual measures of output (value of shipments) and inputs (employment, materials/energy use, capital). This information is aggregated to the firm level. The age of the firm is also available, which is the age of the oldest establishment of the firm. The establishments included in the ASM sampling frame typically have size 20 employees or more, so the parameter estimates are not representative of very small firms. The model yields estimates of ξ, ρ_θ, and σ_θ for young versus old firms at the 4-digit SIC industry level. The estimated values for young firms are then averaged across industries to be used in the calibration of the baseline model. The analysis is limited to the manufacturing sector because of the unavailability of similar data for other sectors of the economy to calculate the revenue-based productivity of an establishment.

A remark is in order for how the estimated parameters of the three-input production function in (1) are used to calibrate the model's two-factor production function. Recall, the model production function is specified as $y_t = \theta_t(k_t^\nu l_t^{1-\nu})^\xi$. In the production function (1) used for estimation, the decreasing returns parameter, ξ, is the same for each of the three inputs. Because the decreasing returns parameter is common to all inputs, in the model's calibration the estimated decreasing returns parameter $\xi = 0.88$ is applied to the two-factor production function. Similarly, the total factor productivity process is not specific to any input (i.e. Hicks neutral) in (1). Therefore, the estimated productivity process based on the three-factor production function in (1) is assumed to apply to the two-factor production in the model.

C Additional Model Results

C.1 Firm Transitions from Entrepreneurial to Corporate Sector

The purpose of this section is to explore the sensitivity of transition probability p as discussed in the calibration section. The notion of transition from the entrepreneurial sector to the corporate sector was introduced only to generate a simple link between the two sectors and allow some entrepreneurial firms to operate without scale and borrowing constraints. The experiments in Table C.1 show that the exact magnitude of the transition rate does not matter substantially in the model's ability to match the data targets, as long as the rate is not too large. Compare the prices and allocations in the baseline economy, where $p = 0.001$, to two stationary equilibria with much higher transition probability values. In all instances, sorting remains broadly consistent with the baseline economy with relatively small changes in how the model matches targeted moments.

C.2 Persistence of Entrepreneurial Ability

The stochastic process for entrepreneurial ability is estimated based on the methodology in Abraham and White (2006). The estimation yields a smaller persistence than what has been found in other studies, such as Decker et al. (2018), Lee and Mukoyama (2015) and Foster et al. (2008).[3] It is of interest to know how the properties of the baseline economy change if a higher value of persistence is used. For this purpose, the model is re-calibrated with a higher value for the quarterly persistence parameter, $\rho_\theta = 0.6$, that is closer to the quarterly transformations of annual values found in other studies. The results are in Table C.2.

The economy with a higher persistence in entrepreneurial ability is able to broadly capture worker sorting as found in the baseline calibration. The new calibration captures most of the targeted moments with the exception of the corporate earnings premium. While the baseline model implies a 16.6% corporate earnings premium, the economy with higher persistence yields a premium of only 9.0%. Overall, there is also a lower degree of worker sorting based on both productivity and assets.

[3] Lee and Mukoyama (2015) does not allow for heterogeneity in the persistence parameter, and Foster et al. (2008) provides estimates for a sample of 11 narrowly defined manufacturing product categories.

Table C.1. The Role of Transition Probability (p)

Variable	**Baseline** ($p = 0.001$)	**No Transitions** ($p = 0.000$)	**Larger Transition Rates** ($p = 0.002$)	($p = 0.004$)
Employment-to-population ratio	82.5%	82.4%	82.7%	83.1%
Share of employment (Entrepreneurial sector)	14.0%	13.2%	13.5%	13.8%
Fraction of entrepreneurs	4.0%	3.5%	5.3%	8.6%
Corporate average earnings premium	16.6%	16.2%	16.6%	14.5%
Wage ratio (w_f/w_e)	1.23	1.23	1.22	1.19
Employment-to-nonemployment (E-to-N) flows	10.3%	10.0%	11.0%	11.0%
Nonemployment-to-employment (N-to-E) flows	39.0%	40.3%	37.3%	32.6%
Job-to-job flows	1.9%	1.9%	1.9%	1.9%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	0.94	0.95	0.96
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.29	1.26	1.19

Notes: Each column represents equilibrium allocations that vary from the baseline by probability of transition to corporate sector. See text for details.

Table C.2. The properties of the model with high persistence of entrepreneurial ideas

Variable	**Baseline** ($\rho_\theta = 0.30$)	**Higher Persistence for θ** ($\rho_\theta = 0.6$)
Employment-to-population ratio	82.5%	83.3%
Share of employment (Entrepreneurial sector)	14.0%	14.0%
Fraction of entrepreneurs	4.0%	4.6%
Corporate average earnings premium	16.6%	9.0%
Wage ratio (w_f/w_e)	1.23	1.10
Employment-to-nonemployment (E-to-N) flows	10.3%	9.0%
Nonemployment-to-employment (N-to-E) flows	39.0%	35.6%
Job-to-job flows	1.9%	2.4%
Ratio of worker productivity (Corporate/Entrepreneurial)	0.95	0.99
Ratio of average worker assets (Corporate/Entrepreneurial)	1.27	1.08
Disutility from labor, α	0.37	0.37
Entrepreneurial sector job offer rate, γ_e	0.07	0.06
Corporate sector job offer rate, γ_f	0.46	0.43
Job-to-job transition rate parameter, $\bar{\gamma}$	0.10	0.11
Entrepreneurial sector job separation rate, ϕ_e	0.05	0.06
Corporate sector job separation rate, ϕ_f	0.10	0.09
Entrepreneurial ability (Mean), $e^{-\mu}$	0.38	0.36

Notes: In the model with a higher persistence for θ, all model parameters are re-estimated to match the data targets.

D Additional Empirical Results

Table D.1. Household net worth by firm size

	Mean		**Quasi-median**	
Firm Size (employees):	**0-49**	**50+**	**0-49**	**50+**
Net worth (All)	$117,500	$125,400	$17,060	$23,590
s.e.	(1,195)	(652)	(287)	(218)
N	53,000	175,000	53,000	175,000
Net worth (Recent hire)	$97,560	$94,440	$10,640	$11,160
s.e.	(1,842)	(1,148)	(310)	(200)
N	19,000	46,000	19,000	46,000
Earnings (All)	$9,856	$13,520	$7,640	$10,380
s.e.	(68)	(339)	(35)	(23)
N	53,000	175,000	53,000	175,000
Earnings (Recent hire)	$7,070	$10,660	$5,133	$6,586
s.e.	(102)	(1,252)	(45)	(40)
N	19,000	46,000	19,000	46,000

Notes: Standard errors in parentheses. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. For quasi-median, the standard errors are calculated using bootstrap.

Table D.2. Regression analysis of household net worth and earnings by firm size

	Estimated Small Firm Coefficient	
Dependent Variable	**OLS**	**Median**
Net worth (All workers)	-0.313^{***}	-0.157^{***}
s.e.	(0.071)	(0.017)
N	228,000	228,000
Net worth (Recent hire)	-0.129	-0.064^{**}
s.e.	(0.105)	(0.034)
N	66,000	66,000
Earnings (All workers)	-0.299^{***}	-0.209^{***}
s.e.	(0.015)	(0.004)
N	228,000	228,000
Earnings (Recent hire)	-0.307^{***}	-0.214^{***}
s.e.	(0.034)	(0.008)
N	66,000	66,000

Notes: Standard errors in parentheses (clustered by time and industry for OLS). All regressions include gender, race, marital status, education level dummies, age, and age-squared, as well as state, industry (2-digit NAICS) and year fixed effects. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. (*), (**), and (***) indicate statistical significance at 10, 5, and 1% levels, respectively.

Table D.3. Regression analysis by firm age with 4-digit NAICS controls

	Estimated Young Firm Coefficient	
Dependent Variable	**OLS**	**Median**
Net worth (All workers)	-0.395^{***}	-0.186^{***}
s.e.	(0.078)	(0.022)
N	$228,000$	$228,000$
Net worth (Recent hire)	-0.150^{*}	-0.057^{**}
s.e.	(0.118)	(0.043)
N	$66,000$	$66,000$
Earnings (All workers)	-0.257^{***}	-0.092^{***}
s.e.	(0.022)	(0.006)
N	$228,000$	$228,000$
Earnings (Recent hire)	-0.197^{***}	-0.080^{***}
s.e.	(0.041)	(0.011)
N	$66,000$	$66,000$

Notes: Standard errors in parentheses (clustered by time and industry for OLS). All regressions include gender, race, marital status, education level dummies, age, and age-squared, as well as state, industry (4-digit NAICS) and year fixed effects. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. (*), (**), and (***) indicate statistical significance at 10, 5, and 1% levels, respectively.

Table D.4. Regression analysis by firm size with 4-digit NAICS controls

	Estimated Small Firm Coefficient	
Dependent Variable	**OLS**	**Median**
Net worth (All workers)	-0.289^{***}	-0.146^{***}
s.e.	(0.065)	(0.018)
N	$228,000$	$228,000$
Net worth (Recent hire)	-0.105	-0.037
s.e.	(0.106)	(0.041)
N	$66,000$	$66,000$
Earnings (All workers)	-0.288^{***}	-0.190^{***}
s.e.	(0.014)	(0.004)
N	$228,000$	$228,000$
Earnings (Recent hire)	-0.311^{***}	-0.337^{***}
s.e.	(0.035)	(0.008)
N	$66,000$	$66,000$

Notes: Standard errors in parentheses (clustered by time and industry for OLS). All regressions include gender, race, marital status, education level dummies, age, and age-squared, as well as state, industry (4-digit NAICS) and year fixed effects. The number of observations N is rounded to prevent disclosure of confidential information. The sample is all SIPP respondents with LEHD employment records. (*), (**), and (***) indicate statistical significance at 10, 5, and 1% levels, respectively.

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